SLLN.

February 19th, 2018.

A sequence of rnd variables:

$\{X_k, \ k=1,2,\dots\}$ i.i.d.

Assume $\mu_X := \mathbb{E}[X_1] < \infty$.

Then,

$$\frac{X_1 + \dots + X_n}{n} \xrightarrow[n\to\infty]{} \mu_X$$

Also: If a function g is such that

$g(X_1)$ is well defined

and $\mathbb{E}[g(X_1)] < \infty$,

then

$$\frac{g(X_1) + \dots + g(X_n)}{n} \xrightarrow[n\to\infty]{} \mathbb{E}[g(X_1)]$$

Risk-neutral Pricing.

$V(T)$ .... payoff of a European derivative security

$\mathbb{P}^*$ ... risk-neutral probab. measure (we get from the replicating portfolio)

$$e^{-rT}\mathbb{E}^*[V(T)] = \underbrace{V(0)}_{\text{Price of the option.}}$$

Monte Carlo Pricing.

Recipe:

- Create simulated stock-price paths.

  From the risk-neutral dist'n.

- Apply the payoff function to the simulated stock-price paths.
  Get: the possible payoffs realized for the stock-price draws

  $$\Rightarrow \quad \frac{v_1 + v_2 + \dots + v_n}{n} =: \bar{v} \qquad n \dots \text{\# of simulations}$$

  is "close to" the expected risk-neutral payoff.

- Finally:

  $e^{-rT}\bar{v}$ ... is the Monte Carlo price.

To increase accuracy by a factor of η, we need to increase the number of variates by η^2.

$$Var[\bar{V}] = \frac{Var[V_1]}{n}$$

The limiting behavior

Define $R(0,T) := \ln\left(\frac{S(T)}{S(0)}\right)$ realized returns.

They will be modeled as normally distributed.

Assume:
- Realized returns over disjoint time periods are independent.
- Realized returns over time intervals of equal length are identically distributed.
- $\boxed{\text{ADDITIVE}}$