W: Feb 27th, 2019.

8. Let $S(t)$ denote the price at time t of a stock that pays no dividends. $\delta = 0$ The Black-Scholes framework holds. Consider a European call option with exercise date T, $T > 0$, and exercise price $S(0)e^{rT}$, where r is the continuously compounded risk-free interest rate.

$K = S(0)e^{rT}$

You are given:

(i) $S(0) = \$100$

(ii) $T = 10$

(iii) $\text{Var}\left[\ln S(t)\right] = 0.4t, \ t > 0$.

Determine the price of the call option.

↳ 1st d_1 & d_2

2nd $N(d_1)$ & $N(d_2)$

3rd $V_c(0) = S(0)e^{-\delta \cdot T} \cdot N(d_1) - Ke^{-rT} \cdot N(d_2)$

(A) \$7.96

(B) \$24.82

(C) \$68.26

(D) \$95.44

(E) There is not enough information to solve the problem.

1.

$1^{\underline{st}}$ $$d_1 = \frac{1}{\sigma\sqrt{T}}\left[\ln\left(\frac{\cancel{S(0)}}{\cancel{S(0)}e^{rT}}\right) + \left(r + \frac{\sigma^2}{2}\right)\cdot T\right]$$

$$d_1 = \frac{1}{\sigma\sqrt{T}}\left[\cancel{-rT} \cancel{+ r\cdot T} + \frac{\sigma^2}{2}\cdot T\right] = \frac{\sigma\sqrt{T}}{2}$$

$$\Rightarrow \quad d_2 = d_1 - \sigma\sqrt{T} = -\frac{\sigma\sqrt{T}}{2}$$

$2^{\underline{nd}}$ $$N(d_1) = N(-d_2) \quad \Rightarrow \quad N(d_2) = 1 - N(d_1)$$

$3^{\underline{rd}}$ $$V_c(0) = S(0)\cdot N(d_1) - S(0)\cancel{e^{rT}}\cdot\cancel{e^{-rT}}\cdot(1 - N(d_1))$$

$$V_c(0) = S(0)\left(N(d_1) - 1 + N(d_1)\right)$$

$$\boxed{V_c(0) = S(0)\left(2N(d_1) - 1\right)}$$

From (iii) : $\mathrm{Var}[\ln(S(t))] = 0.4\cdot t$

w/ $$\ln(S(t)) = \ln(S(0)) + \overbrace{\left(r - \frac{\sigma^2}{2}\right)\cdot t + \sigma\sqrt{t}\cdot Z}^{R(0,t)}$$

$Z \sim N(0,1)$

$$\Rightarrow \mathrm{Var}[\ln(S(t))] = \mathrm{Var}[\sigma\sqrt{t}\cdot Z] = \sigma^2\cdot t$$

$$\Rightarrow \quad \sigma^2 = 0.4$$

2.

In our problem: $$\frac{\sigma\sqrt{T}}{2} = \frac{\sqrt{0.4}\cdot\sqrt{10}}{2} = \frac{\sqrt{4}}{2} = 1$$

$$\Rightarrow V_c(0) = S(0)\left(2\cdot N(1) - 1\right) = 100(0.8413\cdot 2 - 1)$$

$$V_c(0) = 68.26 \ \blacksquare$$

3. You are asked to determine the price of a European put option on a stock. Assuming the Black-Scholes framework holds, you are given:

(i) The stock price is \$100. $S(0) = 100$

(ii) The put option will expire in 6 months. $T = 1/2$

(iii) The strike price is \$98. $K = 98$

(iv) The continuously compounded risk-free interest rate is $r = 0.055$.

(v) $\delta = 0.01$

(vi) $\sigma = 0.50$

Calculate the price of this put option.

(A) \$3.50

(B) \$8.60

(C) \$11.90

(D) \$16.00

(E) \$20.40

$\boxed{1^{st}}$

$$d_1 = \frac{1}{\sigma\sqrt{T}}\left[\ln\left(\frac{S(0)}{K}\right) + \left(r - \delta + \frac{\sigma^2}{2}\right)\cdot T\right]$$

$$d_1 = \frac{1}{0.5\sqrt{\frac{1}{2}}}\left[\ln\left(\frac{100}{98}\right) + \left(0.055 - 0.01 + \frac{0.25}{2}\right)\cdot\frac{1}{2}\right]$$

$$d_1 = 0.298 \approx 0.30$$

$$\Rightarrow d_2 = d_1 - \sigma\sqrt{T} = 0.30 - 0.5\sqrt{\frac{1}{2}}$$

$$d_2 = -0.05599 \approx -0.06$$

$\boxed{2^{nd}}$ $N(-d_1) = N(-0.3) = 1 - N(0.3) =$

$$= 1 - 0.6179 = 0.3821$$

$$N(-d_2) = N(0.06) = 0.5239$$

3.

3rd

$$V_p(0) = Ke^{-rT} N(-d_2) - S(0)e^{-\delta \cdot T} \cdot N(-d_1)$$

$$V_p(0) = 98\,e^{-0.055 \cdot (0.5)} \cdot 0.5239$$

$$- 100\,e^{-0.01(0.5)} \cdot 0.3821 = 11.93$$

$\Rightarrow$ (c) ∎

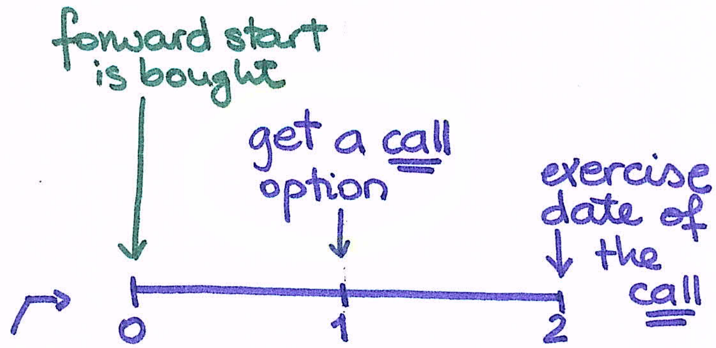
(A) 586

(B) 594

(C) 684

(D) 692

(E) 797



19. Consider a *forward start option* which, 1 year from today, will give its owner a 1-year European call option with a strike price equal to the stock price at that time.

$K = S(1)$

You are given:

(i) The European call option is on a stock that pays no dividends.

(ii) The stock's volatility is 30%. $\sigma = 0.30$

(iii) The forward price for delivery of 1 share of the stock 1 year from today is 100. $F_{0,1}(S) = 100$

(iv) The continuously compounded risk-free interest rate is 8%. $r = 0.08$

Under the Black-Scholes framework, determine the price today of the forward start option.

(A) 11.90

(B) 13.10

(C) 14.50

(D) 15.70

(E) 16.80

5.

At time t such that $t<T$:

$$V_c(t) = S(t)e^{-\delta(T-t)} \cdot N(d_1) - Ke^{-r(T-t)} \cdot N(d_2)$$

w/ $d_1 = \frac{1}{\sigma\sqrt{T-t}}\left[\ln\left(\frac{S(t)}{K}\right) + (r-\delta+\frac{\sigma^2}{2})(T-t)\right]$

and $d_2 = d_1 - \sigma\sqrt{T-t}$

$\Rightarrow$ In our problem:

$$V_c(1) = S(1)\cdot N(d_1) - S(1)\cdot e^{-0.08\cdot 1}\cdot N(d_2)$$

w/ $d_1 = \frac{1}{0.3\sqrt{1}}\left[\cancel{\ln\left(\frac{S(1)}{S(1)}\right)} + (0.08 + \frac{0.09}{2})\cdot 1\right]$

$$d_1 = \frac{0.08 + 0.045}{0.3} \cong 0.42$$

and $d_2 = 0.42 - 0.3 = 0.12$

$$\Rightarrow V_c(1) = S(1)\left(N(d_1) - e^{-0.08}\cdot N(d_2)\right)$$

$$= S(1)\left(0.6628 - e^{-0.08}\cdot 0.5478\right)$$

$$\boxed{V_c(1) = S(1)\cdot 0.1571}$$

At time 0, our forward start option <u>must</u> cost the same as 0.1571 prepaid forward contracts.

$$\Rightarrow V_c(0) = 0.1571\cdot F^P_{0,1}(S)$$

$$= 0.1571\cdot e^{-0.08}\cdot \underbrace{F_{0,1}(S)}_{100} = 14.50$$

6.