Problem 1.1. (5 points) Emmanuel entered an extra special kind of game with his friend Fischer. First, they toss a fair coin. If the coin comes up heads, Emmanuel gives $5,000 to Fischer. If the coin comes up tails, Fischer gives $2,000 to Emmanuel. Then, regardless of the outcome of the first coin toss, they toss the same fair coin again. If it comes up heads, Emmanuel gives Fischer $4,000. If the coin comes up tails, Fischer gives $3,000 to Emmanuel. What is the expected cashflow, i.e., what is the expected amount of money that changes hands and who gives it to whom?

Problem 1.2. (10 pts) For a two-period binomial model, you are given that:

1. each period is one year;
2. the current price of a non-dividend-paying stock $S$ is $S(0) = $20;
3. $u = 1.3$, with $u$ as in the standard notation for the binomial model;
4. $d = 0.9$, with $d$ as in the standard notation for the binomial model;
5. the continuously compounded risk-free interest rate is $r = 0.05$.

Consider a special call option which pays the excess above the strike price $K = 23$ (if any!) at the end of every binomial period. Find the price of this option.

Problem 1.3. (2 points) Which financial instrument will a borrower (at a floating interest rate) acquire if he/she wants to ensure a guaranteed minimal interest rate to be charged on his loan?

Problem 1.4. (5 points) Consider a two-year cap with the cap rate of 0.05. This cap is there to protect the borrower of a floating-rate, three-year, $100,000 loan with interest-only payments at the end of every year. Assume that all the cap payments are made in advance, i.e., at the times when the floating rates are observed.

(i) (2 points) At time $-1$, it is observed that the floating effective annual interest rate equals 0.06. What is the cap payment made at that time?
(ii) (1 points) What is the borrower’s net payment towards the loan repayment at time $-2$?
(iii) (1 points) At time $-2$, it is observed that the floating effective annual interest rate equals 0.045. What is the cap payment made at that time?
(iv) (1 points) What is the borrower’s net payment towards the loan repayment at time $-3$ (excluding the bulk payment of the principal)?
**Problem 1.5.** (5 points) Consider the following binomial interest-rate tree modeling the future evolution of annual *continuously compounded* interest rates. The period-length is one year.

![Interest-Rate Tree](image)

The risk-neutral probability is given to be equal to 1/2. What is the price of a zero-coupon bond redeemable in two years for $1,000?

**Problem 1.6.** (5 points) In our usual notation, consider the following zero-coupon bond prices:

\[
P_0(0, 1) = 0.95, \quad P_0(0, 2) = 0.85, \quad P_0(0, 3) = 0.77, \quad P_0(0, 4) = 0.70.
\]

Calculate the price \( F \) of a forward contract for delivery in two years of a zero-coupon bond with one year to maturity.
Problem 1.7. (5 points) Consider the following binomial interest-rate tree modeling the future evolution of annual continuously compounded interest rates. The period-length is one year.

The risk-neutral probability is given to be equal to 1/2. What is the price of a zero-coupon bond redeemable in three years for $1,000?
Problem 1.8. (13 points) You are tasked with modeling a binomial interest rate tree for effective annual interest rates for the following three years. More precisely, your nodes in the tree need to contain the possible effective annual interest rates for the periods $[0, 1], [1, 2]$ and $[2, 3]$. Today's annual spot rate for the one-year investment is 12%. Your modeling assumption is that at the end of every year, the interest rate can increase (additively) by a factor of 0.04 or decrease (additively) by 0.04.

(i) (3 points) Complete the binomial interest-rate tree, based on the above modeling assumptions.

(ii) (10 points) Assume that the risk-neutral probability of moving up in a single step equals the risk-neutral probability of moving down in a single step in the above tree. Consider a $90$-strike, 2-year European put option on a zero-coupon bond redeemable for $100$ three years from today. What is today’s price of this option?