

UNIVERSITY OF TEXAS AT AUSTIN

HW Assignment 1

The prerequisite material.

Please, provide just the final answer to the True/False and Multiple-choice questions. For the free-response questions, provide your complete solutions if you expect partial credit. For the free-response questions, even if your final answer is correct and your work is not shown, you should not expect **any** partial credit.

Problem 1.1. (10 points)

Provide the definition of an arbitrage portfolio.

Problem 1.2. (2 points) It is never optimal to exercise an American call option on a non-dividend paying stock early. *True or false?*

Problem 1.3. (2 points) Gap put options always have a nonnegative payoff. *True or false?*

Problem 1.4. (6 pts) Let Z be a standard normal random variable. Using the standard normal tables, calculate the following probabilities:

- (i) (2 points) $\mathbb{P}[-1.23 < Z < 2.37]$
- (ii) (2 points) $\mathbb{P}[|Z| < 0.5]$
- (iii) (2 points) $\mathbb{P}[Z^2 > 2.56]$

Problem 1.5. (5 points) An investor short sells one share of a non-dividend-paying stock and buys an at-the-money, T -year, European call option on this stock. The call premium is denoted by $V_C(0)$. Assume that there are no transaction costs. The continuously compounded, risk-free interest rate is denoted by r . Let the argument s represent the stock price at time T .

- (i) (3 points) Determine an algebraic expression for the investor's profit at expiration T in terms of $V_C(0)$, r , T and the strike K .
- (ii) (2 points) In particular, how does the expression you obtained in (i) simplify if the call is in-the-money on the exercise date?

Problem 1.6. (10 pts) For a two-period binomial model, you are given that:

- (1) each period is one year;
- (2) the current price of a non-dividend-paying stock S is $S(0) = \$20$;
- (3) $u = 1.2$, with u as in the standard notation for the binomial model;
- (4) $d = 0.8$, with d as in the standard notation for the binomial model;
- (5) the continuously compounded risk-free interest rate is $r = 0.04$.

Consider a **special** call option which pays the excess above the strike price $K = 23$ (if any!) at the end of **every** binomial period.

Find the price of this option.

Problem 1.7. (5 points) A discrete-dividend-paying stock sells today for \$100 per share. The continuously compounded, risk-free interest rate is 0.04. The first dividend will be paid at in three months in the amount of \$2. The remaining dividends will be equal to \$1 and continue to be paid out quarterly. What is the **prepaid forward price** of this stock for delivery in seven months?

- (a) \$73.02
- (b) \$97.04
- (c) \$98.02
- (d) \$100
- (e) None of the above.

Problem 1.8. (5 points) The random vector (X_1, X_2) is jointly normal. Its marginal distributions are:

$$X_1 \sim N(\text{mean} = 0, \text{variance} = 4), \quad X_2 \sim N(\text{mean} = 1, \text{variance} = 1).$$

The correlation coefficient is given to be

$$\text{corr}[X_1, X_2] = 0.3.$$

What is the variance of the random variable $X = 3X_1 - 2X_2$?

- (a) 32.8
- (b) 47.2
- (c) 54.4
- (d) 58.2
- (e) None of the above.

Problem 1.9. (5 points) A coin is tossed and, independently, a 6-sided die is rolled. Let

$A = \{4 \text{ is obtained on the die}\}$ and

$B = \{Heads \text{ is obtained on the coin and}$
an even number is obtained on the die}.

Then

- (a) A and B are mutually exclusive
- (b) A and B are independent
- (c) $A \subseteq B$
- (d) $A \cap B = B$
- (e) None of the above.