Problem 2.1. (2 pts) Let the stock price $S(t)$ be modeled using the lognormal distribution. Define $Y(t) = S(t)^3$. Then, the random variable $Y(t)$ is lognormal itself. True or false?
Solution: TRUE

Problem 2.2. (2 pts) Let the stochastic process $S = \{S(t), t \geq 0\}$ represent the stock price as in the Black-Scholes model. Let its volatility term be denoted by $\sigma$. Then, the volatility parameter of the process $Y(t) = 2S(t)$ is $4\sigma$.
Solution: FALSE
Note: The correct answer is relevant to the solution for the Sample MFE Problem #54. The volatility parameter of the process $Y$ is $\sigma$.

Problem 2.3. (2 pts)
Call theta may also called time decay. True or false?
Solution: TRUE

Problem 2.4. (5 pts) Which of the following gives the correct values for the delta and gamma of a single share of non-dividend-paying stock?
(a) $\Delta = 1, \Gamma = 1$
(b) $\Delta = 1, \Gamma = 0$
(c) $\Delta = 0, \Gamma = 1$
(d) $\Delta = 0, \Gamma = 0$
(e) None of the above.
Solution: (b)

Problem 2.5. (5 points) Assume the Black-Scholes framework as model for the price of a non-dividend-paying stock. What is the difference between the delta of a European call option and the delta of the otherwise identical put option?
(a) 0
(b) 1
(c) $S(0)$
(d) Not enough information is given to answer this question.
(e) None of the above.
Solution: (b)
Put-call parity.

Problem 2.6. (2 points) Rho measures the sensitivity of a portfolio to the changes in the applicable risk-free interest rate. True or false?
Solution: TRUE
b. (10 pts) Now, set $\delta = 0.001$ and let $V_C(0, T, \delta)$ denote the Black-Scholes European call price for the maturity $T$. Again, how does $V_C(0, T, \delta)$ behave as $T \to \infty$?

c. (4 pts) Interpret in a sentence or two the differences, if there are any, between your answers to questions in a. and b.

Solution:

a. By the Black-Scholes pricing formula, the function $V_C(0, T)$ has the form

$$V_C(0, T) = S(0)N(d_1) - Ke^{-rT}N(d_2),$$

where $N$ denotes the distribution function of the unit normal distribution and

$$d_1 = \frac{1}{\sigma \sqrt{T}} \left[ \ln \left( \frac{S(0)}{K} \right) + \left( r + \frac{1}{2} \sigma^2 \right) T \right],$$

$$d_2 = d_1 - \sigma \sqrt{T}.$$

As $T \to \infty$, we have that

$$d_1 \to \infty \Rightarrow N(d_1) \to 1,$$

$$e^{-rT}N(d_2) \leq e^{-rT} \to 0.$$

Hence,

$$V_C(0, T) \to S(0), \; \text{as} \; T \to \infty.$$  

b. In this case, the price of the call option reads as

$$V_C(0, T, \delta) = S(0)e^{-\delta T}N(d_1) - Ke^{-rT}N(d_2),$$

with

$$d_1 = \frac{1}{\sigma \sqrt{T}} \left[ \ln \left( \frac{S(0)}{K} \right) + \left( r - \delta + \frac{1}{2} \sigma^2 \right) T \right],$$

$$d_2 = d_1 - \sigma \sqrt{T}.$$

Since the function $N$ is bounded between 0 and 1, we see that as $T \to \infty$, $V_C(0, T, \delta) \to 0$.

c. When the stock is paying the dividend, the benefit of owning the stock and opposed to owning the option on that stock lies precisely in the value of the issued dividend. As we can see from above, even a very small dividend yield is going to render the call options for very long maturities worthless.