UNIVERSITY OF TEXAS AT AUSTIN

HW Assignment 4

More on option Greeks. Implied volatility.

Provide your final answer only to the True/False problem(s) and your complete solution to the free-response problems(s). Thank you!

Problem 4.1. (2 points) Call *theta* may also be called time decay. *True or false?*

Problem 4.2. (2 points) Consider a European call and an otherwise identical put. Then, the call vega is strictly greater than the put vega. *True or false?*

Problem 4.3. (2 points) Assume the Black-Scholes stock-price model for the underlying asset of a gap call and an ordinary European call with the strike equal to the trigger of the gap call option. The gap call has the trigger price strictly larger than the strike price. Then, the delta of the gap call is always smaller than the delta of the ordinary call with the same strike price. *True or false?*

Problem 4.4. (2 points) The theta of a European put is always negative. *True or false?*

4.1. **Implied volatility.**

Problem 4.5. (2 points) When implied volatility is calculated for observed option prices with vaying strikes and all other inputs kept the same, then the same parameter σ is obtained regardless of the choice of the strike. *True or false?*

Problem 4.6. (10 points) Let $\mathbf{S} = \{S(t), t \geq 0\}$ denote the price of a non-dividend-paying stock. The stock price today equals \$100. Assume that the Black-Scholes setting holds.

Let r denote the continuously compounded risk-free interest rate.

Consider a European call option with exercise date $T = 10$ and strike price $K = S(0)e^{rT}$. You are given that its price today equals $V_C(0) = \$68.26$.

The goal of this problem is to obtain the implied volatility of the stock S.

(i) (5 pts) Write down the expression for the Black-Scholes price of the European call.
(ii) (3 pts) Simplify the expression you obtained in part (i) so that the call price depends only on the volatility σ.
(iii) (2 pts) Using the properties of the standard normal cumulative distribution function N, the standard normal table, the European call price given in the problem and your answer to part (ii), solve for σ.

Problem 4.7. (10 points) Let $\mathbf{S} = \{S(t), t \geq 0\}$ denote the price of a continuous-dividend-paying stock. The prepaid forward price for delivery of one share of this stock in one year equals \$98.02. Assume that the Black-Scholes model is used for the evolution of the stock price.

Consider a European call and a European put option both with exercise date in one year. They have the same strike price and the same Black-Scholes price equal to \$9.37. What is the implied volatility of the underlying stock?

Problem 4.8. (10 points) Let $\mathbf{S} = \{S(t),\, t \geq 0\}$ denote the price of a non-dividend-paying stock. The current stock price is \$50. Assume that the Black-Scholes model is used for the evolution of the stock price.

Let the continuously compounded, risk-free interest rate be equal to 0.05.

Consider a European call option on this stock with exercise date in one quarter-year and with the strike price equal to $K = 50e^{0.0125}$. The price of this option is observed to be \$3.98. What is the stock's implied volatility?

Problem 4.9. (10 points) Let $\mathbf{S} = \{S(t),\, t \geq 0\}$ denote the price of a continuous-dividend-paying stock. The current stock price is \$100 and its dividend yield is 0.01. Assume that the Black-Scholes model is used for the evolution of the stock price.

Let the continuously compounded, risk-free interest rate be equal to 0.025.

Consider a European call option on this stock with exercise date in nine months and with the strike price equal to $K = 100e^{0.01125}$. The price of this option is observed to be \$10.26. What is the stock's implied volatility?