01/22/2020.

Prereq in-term.

- Basics of options covered in M339D.
- Binomial option pricing.
- Normal dist'n.
- Covariance formula.

- McDonald: "Derivatives Markets" (3rd ed)
- Berk/DeMarzo: "Corporate finance"

January 22^{nd}, 2020.

Binomial asset pricing [review].

T... time horizon

n... the number of periods

$\Rightarrow$ length of every period : $h = \frac{T}{n}$

$$
\begin{tikzcd}[column sep=large]
 & & S_{uu} = u^2 \cdot S(0) \\
 & S_u = u \cdot S(0) \arrow[ur] \arrow[dr] & \\
S(0) \arrow[ur] \arrow[dr] & & S_{ud} = u \cdot d \cdot S(0) \\
 & S_d = d \cdot S(0) \arrow[ur] \arrow[dr] & \\
 & & S_{dd} = d^2 S(0)
\end{tikzcd}
$$

$$\underbrace{\qquad\qquad}_{h}$$

No-arbitrage if $\boxed{d < e^{(r-\delta)h} < u}$

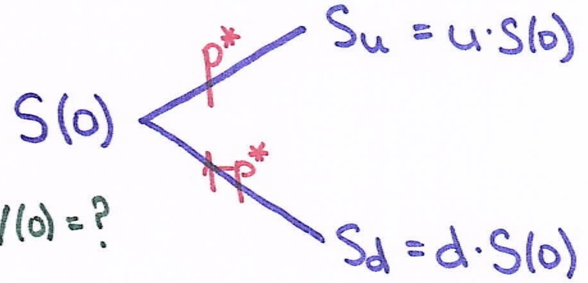
Forward Tree

σ... VOLATILITY

Define: $u = e^{(r-\delta)h + \sigma\sqrt{h}}$

$d = e^{(r-\delta)h - \sigma\sqrt{h}}$

Binomial option pricing: one period [review].



$V(0) = ?$

Payoff

$V_u = v(S_u)$

$V_d = v(S_d)$

Pricing a European option w/ payoff f'tion $v(\cdot)$

This option can be replicated w/ the portfolio w/

- $\Delta = e^{-\delta h} \cdot \dfrac{V_u - V_d}{S_u - S_d}$ shares of stock

and

- $B = e^{-rh} \cdot \dfrac{u \cdot V_d - d \cdot V_u}{u-d}$ @ the risk-free interest rate

$$V(0) = \Delta \cdot S(0) + B = \dots = e^{-rT} \cdot [p^* \cdot V_u + (1-p^*) \cdot V_d]$$

RISK-NEUTRAL PRICE

w/ $p^* = \dfrac{e^{(r-\delta)h} - d}{u-d}$ ... the risk-neutral probability