W: January 27th, 2020.

Binomial pricing of currency options.

- underlying : the foreign currency (FC)
  w/ r_F ... continuously compounded, risk-free i.r. for FC

... as opposed to our domestic currency (DC)
w/ r_D ... ccrfir for DC

$x(t)$, $t \geq 0$ ... exchange rate from FC to DC

Recall:

- $F^P_{0,T}(x) = e^{-r_F \cdot T} \cdot x(0)$
- $F_{0,T}(x) = e^{(r_D - r_F)\cdot T} \cdot x(0)$
- Put-Call Parity:

$$V_C(0) - V_P(0) = F^P_{0,T}(x) - Ke^{-r_D \cdot T}$$
$$= x(0)e^{-r_F \cdot T} - Ke^{-r_D \cdot T}$$

Analogy:

Foreign currency $\longleftrightarrow$ continuous-dividend stocks

$x(t), t \geq 0 \longleftrightarrow S(t), t \geq 0$

r_F ... ccrfir for FC $\longleftrightarrow$ δ ... dividend yield

Q: How can we adapt this analogy to binomial option pricing?

$\boxed{\text{ONE PERIOD}}$

$$\begin{tikzcd}[column sep=large]
 & x_u = u\cdot x(0) \\
x(0) \arrow[ur, "p^*"] \arrow[dr, "1-p^*"'] & \\
 & x_d = d\cdot x(0)
\end{tikzcd}$$

$$\underbrace{\qquad\qquad}_{h = T}$$

PAYOFF		REPLICATING PORT.
$V_u = v(x_u)$	$=$	$\Delta e^{r_F \cdot T} \cdot x_u + B e^{r_D \cdot T}$
$V_d = v(x_d)$	$=$	$\Delta e^{r_F \cdot T} \cdot x_d + B e^{r_D \cdot T}$

$\Downarrow$

Complete analogy!

The replicating portfolio:

- Δ ... # of units of FC bought @ time 0 and deposited to earn r_F for the duration of the period
- B ... the risk-free investment in the DC

In particular:

$$p^* = \frac{e^{(r_D - r_F)\cdot h} - d}{u - d} \quad \text{... DEF'N}$$

4. For a two-period binomial model, you are given:

(i) Each period is one year.

(ii) The current price for a nondividend-paying stock is 20.

(iii) $u = 1.2840$, where u is one plus the rate of capital gain on the stock per period if the stock price goes up.

(iv) $d = 0.8607$, where d is one plus the rate of capital loss on the stock per period if the stock price goes down.

(v) The continuously compounded risk-free interest rate is 5%.

Calculate the price of an American call option on the stock with a strike price of 22.

(A) 0
(B) 1
(C) 2
(D) 3
(E) 4

$T = 3/4$ $\overbrace{\text{dollar-denominated}}^{\$\ldots DC}$ £... FC

✗ **5.** Consider a 9-month dollar-denominated American put option on British pounds. You are given that:

(i) The current exchange rate is 1.43 US dollars per pound. $x(0) = 1.43$

(ii) The strike price of the put is 1.56 US dollars per pound. $K = 1.56$

(iii) The volatility of the exchange rate is $\sigma = 0.3$.

(iv) The US dollar continuously compounded risk-free interest rate is 8%. $r_{\$} = r_D = 0.08$

(v) The British pound continuously compounded risk-free interest rate is 9%. $r_{£} = r_F = 0.09$

$n = 3$

Using a three-period binomial model, calculate the price of the put.

(A) 0.23
(B) 0.25
(C) 0.27
(D) 0.29
(E) 0.31

$\Rightarrow h = 1/4$ ... length of each period

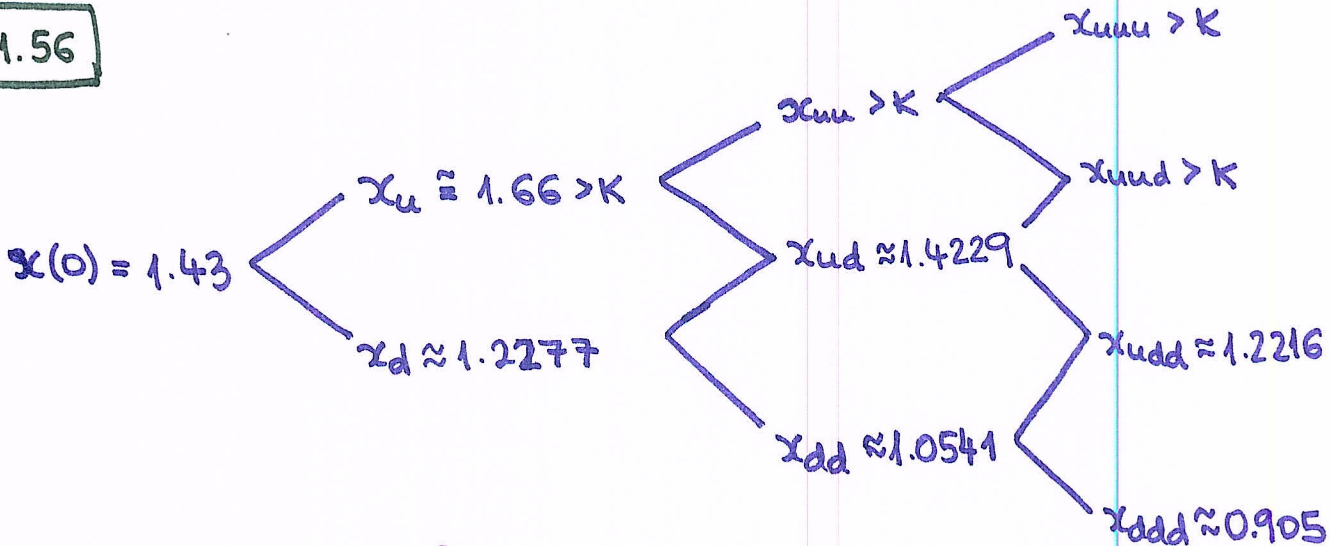
The **forward** tree is the default tree!

$$p^* = \frac{1}{1+e^{\sigma\sqrt{h}}} = \frac{1}{1+e^{0.15}} \approx 0.46257$$

$$u = e^{(r_D - r_F)\cdot h + \sigma\sqrt{h}} = e^{(0.08-0.09)(0.25)+0.15} \approx 1.1589$$

$$d = e^{(r_D - r_F)\cdot h - \sigma\sqrt{h}} = e^{(-0.01)(0.25) - 0.15} \approx 0.8585$$

PUT $\boxed{K = 1.56}$



⋮

Finish @ home :)

Binomial option pricing: Futures options.

Futures contracts: tradable versions of forward contracts

- liquid counterparts of forward contracts w/ observeable prices

$\Rightarrow$ we can define calls/puts w/ futures contracts as the underlying

0 — T — T_F ... the delivery date of the futures contract

T: the option's expiration date

Analogy:

Futures $\longleftrightarrow$ continuous-dividend stocks

r ... continuously-compounded risk-free interest rate $\longleftrightarrow$ δ ... dividend yield