$\boxed{\text{W: March } 1^{st}, \text{ 2019.}}$

33. You own one share of a nondividend-paying stock. Because you worry that its price may drop over the next year, you decide to employ a *rolling insurance strategy*, which entails obtaining one 3-month European put option on the stock every three months, with the first one being bought immediately.

You are given:

(i) The continuously compounded risk-free interest rate is 8%.

(ii) The stock's volatility is 30%.

(iii) The current stock price is 45.

(iv) The strike price for each option is 90% of the then-current stock price.

Your broker will sell you the four options but will charge you for their total cost now.

Under the Black-Scholes framework, how much do you now pay your broker?

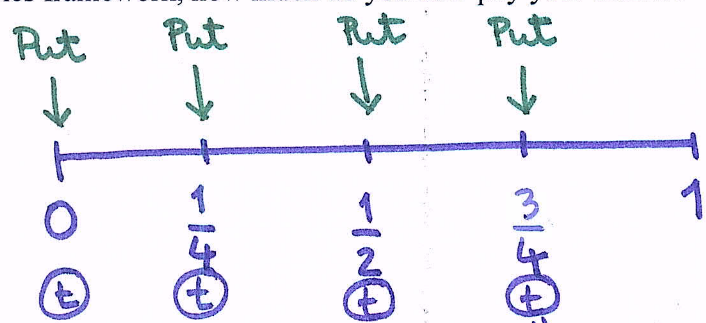
(A) 1.59

(B) 2.24

(C) 2.86

(D) .48

(E) 3.61



For every put in the rolling insurance, there is a quarter-year to exercise date & $\boxed{K = 0.9 \cdot S(t)}$

For every t @ which a put option is received:

$$d_1(t) = \frac{1}{\sigma\sqrt{1/4}}\left[\ln\left(\frac{\cancel{S(t)}}{0.9\cdot\cancel{S(t)}}\right) + \left(0.08 + \frac{0.09}{2}\right)\cdot\frac{1}{4}\right]$$

$$d_1(t) = \frac{1}{0.3(0.5)}\left[-\ln(0.90) + 0.125 \cdot 0.25\right]$$

$d_1(t) = 0.9107 \approx 0.91$ for every t

1.

$$\Rightarrow d_2(t) = d_1(t) - \sigma\sqrt{\underbrace{1/4}}$$

time to expiration for each of our puts

$$d_2(t) = 0.9107 - 0.30 \cdot \frac{1}{2} = 0.7607 \approx 0.76$$

$$N(-d_1) = N(-0.91) = 1 - N(0.91) = 1 - 0.8186 = 0.1814$$

$$N(-d_2) = N(-0.76) = 1 - N(0.76) = 1 - 0.7764 = 0.2236$$

$$\Rightarrow V_P(t) = \overbrace{0.9 \cdot S(t)}^{\text{strike price}} \cdot e^{-0.08(1/4)} \cdot 0.2236$$

$$- S(t) \cdot 0.1814$$

$$V_P(t) = S(t)\left[0.9e^{-0.02} \cdot 0.2236 - 0.1814\right]$$

$$= S(t) \cdot 0.01586$$

$\Rightarrow$ For every issuance date $t = 0, 1/4, 1/2, 3/4$, today's worth of the put option obtained on that date is

$$F^P_{0,t}(S) \cdot 0.01586$$

rounding

The stock pays no dividends, so $F^P_{0,t}(S) = S(0)$.

$\Rightarrow$ Altogether, the time 0 price of the rolling insurance strategy is

Prometric vs. tables.

$$4 \cdot 45 \cdot 0.01586 = 2.854 \approx 2.86$$

2.

\# of puts

S(0)

Gap Options.

K_t ... trigger price

K_s ... strike price

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<u>Gap call:</u> $V_{GC}(T) = (S(T) - K_s) \cdot \mathbb{I}_{[S(T) > K_t]}$

<u>Gap put:</u> $V_{GP}(T) = (K_s - S(T)) \cdot \mathbb{I}_{[S(T) < K_t]}$

$\Rightarrow$ In the Black-Scholes model, we have (using the same argument as for vanilla calls & puts):

$$V_{GC}(0) = S(0)e^{-\delta \cdot T} \cdot N(d_1) - K_s e^{-rT} \cdot N(d_2)$$

and

$$V_{GP}(0) = K_s e^{-rT} N(-d_2) - S(0)e^{-\delta \cdot T} \cdot N(-d_1)$$

w/

$$d_1 = \frac{1}{\sigma\sqrt{T}} \left[\ln\left(\frac{S(0)}{K_t}\right) + \left(r - \delta + \frac{\sigma^2}{2}\right) \cdot T \right]$$

and

$$d_2 = d_1 - \sigma\sqrt{T}$$

$\delta = 0$

Problem 2.8. (15 points) The price of a non-dividend-paying stock is modeled using the Black-Scholes framework. Today's stock price is equal to \$100 and its volatility is 0.2.

The continuously-compounded, risk-free interest rate equals 0.04.

You are constructing a **zero-cost** gap put option. The option is supposed to pay $K - S(1/4)$ in three months if the condition $S(1/4) < \underbrace{110}_{K_t \ldots \text{trigger price}}$ is satisfied. Find the strike price K of your gap put option such that the gap put is free.

1^{st} $$d_1 = \frac{1}{0.2\sqrt{1/4}}\left[\ln\left(\frac{100}{110}\right) + \left(0.04 + \frac{(0.2)^2}{2}\right)\cdot\frac{1}{4}\right]$$

$$d_1 = \frac{1}{0.2\cdot\frac{1}{2}}\left[\ln\left(\frac{10}{11}\right) + (0.04 + 0.02)\cdot\frac{1}{4}\right]$$

$$d_1 = -0.80$$

$$\Rightarrow \quad d_2 = d_1 - \sigma\sqrt{T} = -0.80 - 0.10 = -0.90$$

2^{nd} $$N(-d_1) = N(0.8) = 0.7881,$$
$$N(-d_2) = N(0.9) = 0.8159.$$

3^{rd} For our gap put, we have

$$V_{GP}(0) = K_s e^{-r\cdot T} N(-d_2) - S(0)\cdot N(-d_1) = 0$$

we're looking for a zero-cost gap put

$$\Rightarrow \quad K_s = \frac{100\,(0.7881)}{e^{-0.04\cdot(1/4)}\,(0.8159)} = 97.5635$$

4.

Black·Scholes "Master" Formula.

So far: Vanilla calls & puts on continuous·dividend·paying stocks:

$$\begin{cases} V_C(0) = \overbrace{S(0)e^{-\delta\cdot T}}^{=:F^P_{0,T}(S)} N(d_1) - \overbrace{Ke^{-rT}}^{=:PV_{0,T}(K) = F^P_{0,T}(K)} N(d_2) \\ V_P(0) = Ke^{-rT} N(-d_2) - S(0)e^{-\delta T} N(-d_1) \end{cases}$$

w/ $d_1 = \frac{1}{\sigma\sqrt{T}}\left[\ln\left(\frac{S(0)}{K}\right) + (r - \delta + \frac{\sigma^2}{2})\cdot T\right]$

and

$d_2 = d_1 - \sigma\sqrt{T}$

Note:

$$d_1 = \frac{1}{\sigma\sqrt{T}}\left[\ln\left(\frac{S(0)e^{-\delta T}}{Ke^{-rT}}\right) + \frac{\sigma^2 \cdot T}{2}\right]$$

$\Rightarrow$

$$\begin{cases} V_C(0) = F^P_{0,T}(S)\cdot N(d_1) - F^P_{0,T}(K)\cdot N(d_2) \\ V_P(0) = F^P_{0,T}(K)\cdot N(-d_2) - F^P_{0,T}(S)\cdot N(-d_1) \end{cases}$$

w/ $d_1 = \frac{1}{\sigma\sqrt{T}}\left[\ln\left(\frac{F^P_{0,T}(S)}{F^P_{0,T}(K)}\right) + \frac{\sigma^2 \cdot T}{2}\right]$

and

$d_2 = d_1 - \sigma\sqrt{T}$

For review: In the B·S model,

$$\circledast \quad S(T) = S(0)\, e^{(\alpha - \delta - \frac{\sigma^2}{2})\cdot T + \sigma\sqrt{T}\cdot Z} \qquad \text{w/ } Z \sim N(0,1)$$

$$F^{P}_{t,T_F}(S) = \underline{\underline{S(t)}}\, e^{-\delta(T_F - t)}$$

↑ delivery date

valuation date

$$\Rightarrow \quad F^{P}_{t,T_F}(S) = \underline{\underline{S(0)\, e^{(\alpha - \delta - \frac{\sigma^2}{2})\cdot t + \sigma\sqrt{t}\cdot Z}}} \cdot e^{-\delta(T_F - t)}$$

also lognormal