Black·Scholes : Currency Options

W: March 4th, 2019.

Recall: * domestic currency : DC

- r_D ... ccrfir

* foreign currency : FC ... underlying asset

- r_F ... ccrfir

$x(t), t \geq 0$ ... exchange rate from the FC to the DC

Prepaid forward: $F_{0,T}^P(x) = x(0) \cdot e^{-r_F \cdot T}$

Put·call Parity:

DC-denominated

$$V_C(0) - V_P(0) = F_{0,T}^P(x) - PV_{0,T}(K)$$

$$= x(0)e^{-r_F \cdot T} - Ke^{-r_D \cdot T}$$

Compare the above to continuous·dividend·paying stocks:

- $F_{0,T}^P(S) = S(0)e^{-\delta \cdot T}$
- $V_C(0) - V_P(0) = S(0)e^{-\delta \cdot T} - Ke^{-r \cdot T}$

We used the analogy:

$r \longleftrightarrow r_D$ ... DC ccrfir

$\delta \longleftrightarrow r_F$ ... FC ccrfir

* Garman-Kohlhagen model:

Black-Scholes analogue for foreign currencies *

=> the call/put prices will be:

$$V_c(0) = x(0)\,e^{-r_F \cdot T} \cdot N(d_1) - Ke^{-r_D \cdot T} \cdot N(d_2)$$

$$V_p(0) = Ke^{-r_D \cdot T} N(-d_2) - x(0)\,e^{-r_F \cdot T} \cdot N(-d_1)$$

w/ $d_1 = \frac{1}{\sigma\sqrt{T}}\left[\ln\left(\frac{x(0)}{K}\right) + \left(r_D - r_F + \frac{\sigma^2}{2}\right)\cdot T\right]$

and $d_2 = d_1 - \sigma\sqrt{T}$

6. You are considering the purchase of 100 units of a 3-month 25-strike European call option on a stock.

You are given:

(i) The Black-Scholes framework holds.
(ii) The stock is currently selling for 20.
(iii) The stock's volatility is 24%.
(iv) The stock pays dividends continuously at a rate proportional to its price. The dividend yield is 3%.
(v) The continuously compounded risk-free interest rate is 5%.

Calculate the price of the block of 100 options.

(A) 0.04
(B) 1.93
(C) 3.63
(D) 4.22
(E) 5.09

3.

7. Company A is a U.S. international company, and Company B is a Japanese local company. Company A is negotiating with Company B to sell its operation in Tokyo to Company B. The deal will be settled in Japanese yen. To avoid a loss at the time when the deal is closed due to a sudden devaluation of yen relative to dollar, Company A has decided to buy at-the-money dollar-denominated yen put of the European type to hedge this risk.

$K = x(0)$ Right to sell the Japanese ¥.

You are given the following information:

(i) The deal will be closed 3 months from now. $T = 1/4$
(ii) The sale price of the Tokyo operation has been settled at 120 billion Japanese yen. <=> to hedge: buy $120 \cdot 10^9$ puts on ¥
(iii) The continuously compounded risk-free interest rate in the U.S. is 3.5%. $r_D = r_\$ = 0.035$
(iv) The continuously compounded risk-free interest rate in Japan is 1.5%. $r_F = r_¥ = 0.015$
(v) The current exchange rate is 1 U.S. dollar = 120 Japanese yen. $x(0) = \frac{1}{120}$ [$/¥]
(vi) The daily volatility of the yen per dollar exchange rate is 0.261712%.
(vii) 1 year = 365 days; 3 months = ¼ year.

Calculate Company A's option cost.

σ_h w/ $h = \frac{1}{365}$

$\sigma_h = \sigma\sqrt{h} = \sigma \cdot \sqrt{\frac{1}{365}}$

$\Rightarrow \sigma = \frac{0.261712}{100} \cdot \sqrt{365}$

$\Rightarrow \boxed{\sigma = 0.05}$

$1^{\underline{\underline{st}}}$ $d_1 = \frac{1}{\sigma\sqrt{T}}\left[\cancel{\ln\left(\frac{x(0)}{K}\right)} + \left(r_D - r_F + \frac{\sigma^2}{2}\right)\cdot T\right]$

at the money

$$d_1 = \frac{0.035 - 0.015 + \frac{(0.05)^2}{2}}{0.05}\sqrt{1/4} = 0.2125 \approx 0.21$$

$$\Rightarrow \quad d_2 = d_1 - \sigma\sqrt{T} = 0.2125 - 0.05\cdot 0.5$$
$$= 0.1875 \approx 0.19$$

$2^{\underline{\underline{nd}}}$ $N(-d_1) = 1 - N(0.21) = 1 - 0.5832 = 0.4168,$

$N(-d_2) = 1 - N(0.19) = 1 - 0.5753 = 0.4247.$

$3^{\underline{\underline{rd}}}$

$$V_p(0) = \frac{1}{120} e^{-0.035(1/4)}\cdot 0.4247$$
$$- \frac{1}{120} e^{-0.015(1/4)}\cdot 0.4168$$

$\Rightarrow$ answer: The total cost of the hedge

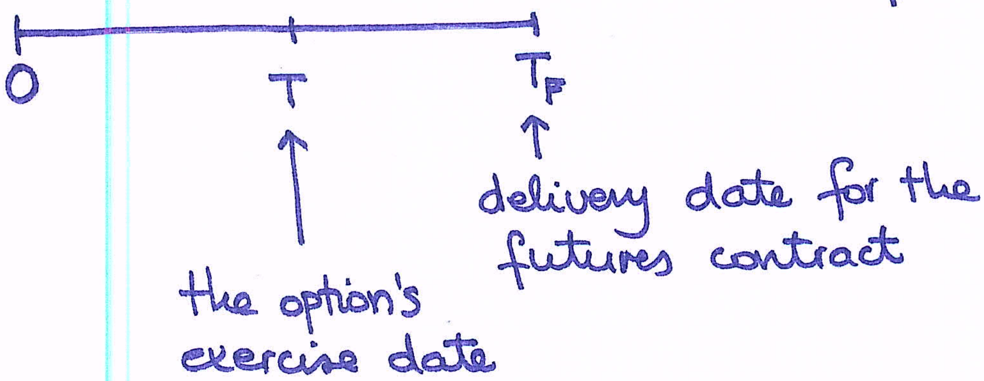
$$120\cdot 10^9\cdot V_p(0) = 10^9\left(e^{-0.035(1/4)}\cdot 0.4247 - e^{-0.015(1/4)}\cdot 0.4168\right)$$

$$\cong 5.7\times 10^6$$

At home: Try to resolve the problem using the Prometric calculator :)

Black·Scholes : Futures options

$T \leq T_F$



We use the same analogy: $\boxed{\delta \longleftrightarrow r}$

Black Formula

$$V_c(0) = F_{0,T_F} \cdot e^{-r \cdot T} \cdot N(d_1) - K e^{-rT} \cdot N(d_2)$$

$$V_c(0) = e^{-r \cdot T} \left(F_{0,T_F} \cdot N(d_1) - K \cdot N(d_2) \right)$$

and

$$V_p(0) = e^{-r \cdot T} \left(K \cdot N(-d_2) - F_{0,T_F} \cdot N(-d_1) \right)$$

w/

$$d_1 = \frac{1}{\sigma \sqrt{T}} \left[\ln\left(\frac{F_{0,T_F}}{K} \right) + \left(\cancel{r} - \cancel{r} + \frac{\sigma^2}{2} \right) \cdot T \right]$$

and

$$d_2 = d_1 - \sigma \sqrt{T}$$

Q: What if we have call w/ $T_F = T$? 5.

This is just equivalent to a vanilla call on the underlying asset.

Q: What if the futures option is $\underbrace{\text{at the money}}$?

$$\Rightarrow d_1 = \frac{1}{\sigma\sqrt{T}}\left[\cancel{\ln\left(\frac{F_{0,T_F}}{K}\right)} + \frac{\sigma^2 \cdot T}{2}\right] = \frac{\sigma\sqrt{T}}{2}$$

and $d_2 = d_1 - \sigma\sqrt{T} = -\frac{\sigma\sqrt{T}}{2}$

$$\Rightarrow V_c(0) = e^{-r \cdot T} \cdot F_{0,T_F} \cdot \left(N\left(\frac{\sigma\sqrt{T}}{2}\right) - N\left(-\frac{\sigma\sqrt{T}}{2}\right)\right)$$

$$V_c(0) = e^{-r \cdot T} \cdot F_{0,T_F} \left(2 \cdot N\left(\frac{\sigma\sqrt{T}}{2}\right) - 1\right) = V_p(0)$$

(A) 0.67

(B) 1.12

(C) 1.49

(D) 5.18

(E) 7.86

55. Assume the Black-Scholes framework. Consider a 9-month at-the-money European put option on a futures contract. You are given:

(i) The continuously compounded risk-free interest rate is 10%.

(ii) The strike price of the option is 20. $= K$

(iii) The price of the put option is 1.625.

If three months later the futures price is 17.7, what is the price of the put option at that time?

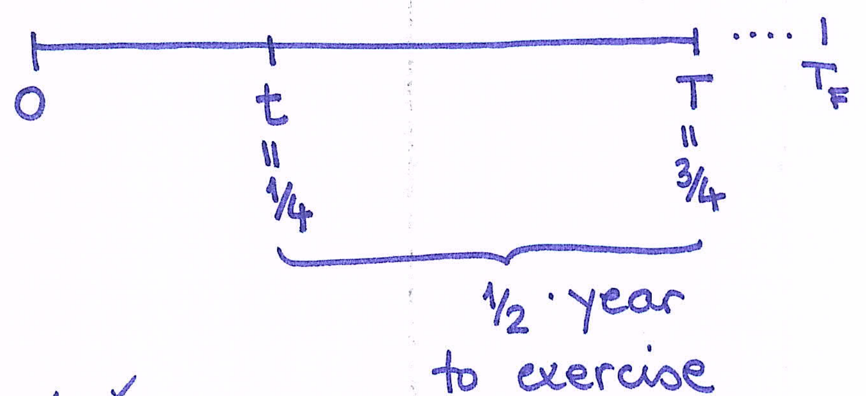
(A) 2.09

(B) 2.25

(C) 2.45

(D) 2.66

(E) 2.83



$$V_P(t) = K e^{-r(T-t)} \cdot N(-d_2(t)) - F_{t,T_F}\, e^{-r(T-t)} \cdot N(-d_1(t))$$

We need σ for $d_1(t)$ and $d_2(t)$

We get it first from the PUT price @ time-0.

7.