(A) 0.67

(B) 1.12

(C) 1.49

(D) 5.18

(E) 7.86

$\boxed{\text{W: March } 6^{th}, 2019.}$

$T = 3/4$

55. Assume the Black-Scholes framework. Consider a 9-month at-the-money European put option on a futures contract. You are given:

(i) The continuously compounded risk-free interest rate is 10%.

(ii) The strike price of the option is 20.

(iii) The price of the put option is 1.625.

If three months later the futures price is 17.7, what is the price of the put option at that time?

(A) 2.09

(B) 2.25

(C) 2.45

(D) 2.66

(E) 2.83

Focusing on the time-0 price!

$$d_1(0) = \frac{1}{\sigma\sqrt{T}}\left[\cancel{\ln\left(\frac{F_{0,T}(F)}{K}\right)} + \frac{\sigma^2 \cdot T}{2}\right]$$

at-the-money

$$\Rightarrow d_1(0) = \frac{\sigma\sqrt{T}}{2}$$

$$\Rightarrow d_2(0) = d_1(0) - \sigma\sqrt{T} = -\frac{\sigma\sqrt{T}}{2}$$

So, $V_P(0) = e^{-r\cdot T}\cdot K\cdot(N(-d_2) - N(-d_1))$

$$= e^{-r\cdot T}\cdot K\cdot\left(N\left(\frac{\sigma\sqrt{T}}{2}\right) - N\left(-\frac{\sigma\sqrt{T}}{2}\right)\right)$$

$$V_P(0) = Ke^{-r\cdot T}\left(2N\left(\frac{\sigma\sqrt{T}}{2}\right) - 1\right)$$

$\Rightarrow$ In our problem:

$$1.625 = 20e^{-0.10(3/4)}\left(2\cdot N\left(\frac{\sigma\sqrt{T}}{2}\right) - 1\right)$$

$$2N\left(\frac{\sigma\sqrt{T}}{2}\right) = \frac{1.625}{20}e^{0.10(3/4)} + 1 \qquad (1.)$$

$$N\left(\frac{\sigma\sqrt{T}}{2}\right) = \frac{1}{2}\left(\frac{1.625}{20}e^{0.075} + 1\right) = 0.5438$$

From the std normal tables:

$$\frac{\sigma\sqrt{T}}{2} = 0.11$$

$$\sigma = \frac{0.22}{\frac{\sqrt{3}}{2}} = 0.254$$

We reprice the option @ time $\frac{1}{4}$:

$$0 \qquad \underbrace{t = 1/4 \qquad\qquad T = 3/4}_{T-t = 1/2}$$

$$d_1(1/4) = \frac{1}{0.254\sqrt{1/2}} \cdot \left[\ln\left(\frac{17.7}{20}\right) + \frac{(0.254)^2}{2} \cdot \frac{1}{2} \right]$$

$$= -0.59$$

$$\Rightarrow d_2(1/4) = d_1(1/4) - \sigma\sqrt{T-t} = -0.59 - 0.254\sqrt{1/2}$$

$$= -0.77$$

$$\Rightarrow N(-d_1(1/4)) = N(0.59) = 0.7224$$

$$N(-d_2(1/4)) = N(0.77) = 0.7794$$

$$\Rightarrow V_p(1/4) = e^{-r(T-t)}\left(K \cdot N(-d_2(1/4)) - F_{t,T_F} \cdot N(-d_1(1/4))\right)$$

$$= e^{-0.10 \cdot (1/2)} \cdot (20 \cdot 0.7794 - 17.7 \cdot 0.7224)$$

$$= 2.661 \Rightarrow (\otimes)$$

Black·Scholes: Discrete·dividend·paying stocks.

Recall: "Master" B·S formula

$$V_c(0) = F^P_{0,T}(S)\cdot N(d_1) - F^P_{0,T}(K)\cdot N(d_2)$$

$$V_p(0) = F^P_{0,T}(K)\cdot N(-d_2) - F^P_{0,T}(S)\cdot N(-d_1)$$

w/ $d_1 = \frac{1}{\sigma\sqrt{T}}\left[\ln\left(\frac{F^P_{0,T}(S)}{F^P_{0,T}(K)}\right) + \frac{\sigma^2 \cdot T}{2}\right]$

and $d_2 = d_1 - \sigma\sqrt{T}$

For discrete dividends:

$$F^P_{t,T}(S) = S(t) - \sum_{t<s\leq T} PV_{t,s}(D_s)$$

w/ s... dividend time
D_s... amt of dividend

We are really using the Black·Scholes model for $F^P_{t,T_F}(S)$.
Its volatility is now σ.

$T = 1/2$

15. For a six-month European put option on a stock, you are given:

(i) The strike price is \$50.00. $K = 50$

(ii) The current stock price is \$50.00. $S(0) = 50$

(iii) The only dividend during this time period is \$1.50 to be paid in four months.

$D = 1.50$ $t_D = 1/3$

(iv) $\boxed{\sigma = 0.30}$

(v) The continuously compounded risk-free interest rate is 5% $r = 0.05$

Under the Black-Scholes framework, calculate the price of the put option.

store this!

(A) \$3.50

(B) \$3.95

(C) \$4.19

(D) \$4.73

(E) \$4.93

$$F^P_{0,1/2}(S) = 50 - 1.50\, e^{-0.05(1/3)} = 48.52$$

$$d_1 = \frac{1}{\sigma\sqrt{T}}\left[\ln\left(\frac{F^P_{0,T}(S)}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)\cdot T\right]$$

$$\ln\left(\frac{F^P_{0,T}(S)}{F^P_{0,T}(K)}\right) = \ln\left(\frac{F^P_{0,T}(S)}{Ke^{-r\cdot T}}\right)$$

$$= \ln\left(\frac{F^P_{0,T}(S)}{K}\right) + \cancel{\ln}(\cancel{e}^{r\cdot T})$$

$$= \ln\left(\frac{F^P_{0,T}(S)}{K}\right) + r\cdot T$$

$$d_1 = \frac{1}{0.3\sqrt{1/2}}\left[\ln\left(\frac{48.52}{50}\right) + \left(0.05 + \frac{0.09}{2}\right)\cdot\frac{1}{2}\right]$$

$$d_1 = 0.0827 \approx 0.08 \Rightarrow d_2 = 0.08 - 0.3\sqrt{1/2} \cong -0.13$$

4.

$N(-d_1) = 1 - N(0.08) = 1 - 0.5319 = 0.4681$

$N(-d_2) = N(0.13) = 0.5517$

$\Rightarrow$

$Y_p(0) = 50e^{-0.025} \cdot 0.5517 - 48.52 \cdot 0.4681 = 4.19$

$\Rightarrow$ (c) ■

$T=1 \quad K=45$

19. Consider a one-year 45-strike European put option on a stock S. You are given:

(i) The current stock price, $S(0)$, is 50.00.

(ii) The only dividend is 5.00 to be paid in nine months. $t_D = 3/4$; $D=5$

(iii) $\mathrm{Var}[\ln F^P_{t,1}(S)] = \underbrace{0.01}_{=\sigma^2} \times t, \quad 0 \le t \le 1.$

$\Rightarrow$ volatility parameter of prepaid forward $\sigma = 0.1$

(iv) The continuously compounded risk-free interest rate is 12%.

Under the Black-Scholes framework, calculate the price of 100 units of the put option.

(A) 1.87

(B) 18.39

(C) 18.69

(D) 19.41

(E) 23.76

$F^P_{0,1}(S) = 50 - 5e^{-0.12\cdot(3/4)} = 45.43$

$d_1 = \frac{1}{0.1\sqrt{1}}\left[\ln\left(\frac{45.43}{45}\right) + \left(0.12 + \frac{0.01}{2}\right)\cdot 1\right]$

$d_1 = 1.345 \approx 1.35$

$\Rightarrow d_2 \cong 1.25$

$N(-d_1) = 1 - N(1.35) = 1 - 0.9115 = 0.0885$

$N(-d_2) = 1 - N(1.25) = 1 - 0.8944 = 0.1056$

$V_P(0) = 45e^{-0.12}\cdot 0.1056 - 45.43 \cdot 0.0885$

$= 0.1941 \Rightarrow (D)$

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