M339W: March 2^{nd}, 2020.

Black·Scholes: Discrete·dividend·paying stocks

$$F^P_{t,T}(S) = S(t) - \sum_{t<u\leq T} PV_{t,u}(D_u)$$

w/ u... the dividend pmt time(s)
and D_u... the amt of the dividend paid @ time u.

We are really using the Black·Scholes model for $F^P_{t,T}(S)$.

Its volatility is now denoted by σ

Recall: The "Master" Black·Scholes formula

$$V_C(0) = F^P_{0,T}(S)\cdot N(d_1) - F^P_{0,T}(K)\cdot N(d_2)$$

$$V_P(0) = F^P_{0,T}(K)\cdot N(-d_2) - F^P_{0,T}(S)\cdot N(-d_1)$$

w/ $$d_1 = \frac{1}{\sigma\sqrt{T}}\left[\ln\left(\frac{F^P_{0,T}(S)}{F^P_{0,T}(K)}\right) + \frac{\sigma^2 \cdot T}{2}\right]$$

and $$d_2 = d_1 - \sigma\sqrt{T}$$

$T = \frac{1}{2}$

15. For a six-month European put option on a stock, you are given:

(i) The strike price is \$50.00. $K = 50$

(ii) The current stock price is \$50.00. $S(0) = 50$

(iii) The only dividend during this time period is \$1.50 to be paid in four months.

$D = 1.50$ $t_D = \frac{1}{3}$

(iv) $\boxed{\sigma = 0.30}$

(v) The continuously compounded risk-free interest rate is 5%. $r = 0.05$

Under the Black-Scholes framework, calculate the price of the put option.

STORE

(A) \$3.50

(B) \$3.95

(C) \$4.19

(D) \$4.73

(E) \$4.93

$$F^P_{0,1/2}(S) = 50 - 1.50\, e^{-0.05(1/3)} = 48.52$$

$$d_1 = \frac{1}{\sigma\sqrt{T}}\left[\ln\left(\frac{F^P_{0,T}(S)}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)\cdot T\right]$$

$$d_1 = \frac{1}{0.3\sqrt{\frac{1}{2}}}\left[\ln\left(\frac{48.52}{50}\right) + \left(0.05 + \frac{0.09}{2}\right)\cdot \frac{1}{2}\right]$$

$$d_1 = 0.0827 \approx 0.08$$

$$\Rightarrow d_2 = 0.08 - 0.3\sqrt{\tfrac{1}{2}} \approx -0.13$$

$$N(-d_1) = 1 - N(0.08) = 1 - 0.5319 = 0.4681$$

$$N(-d_2) = N(0.13) = 0.5517$$

$$\Rightarrow V_P(0) = 50e^{-0.025}(0.5517) - 48.52\,(0.4681)$$

$$V_P(0) = 4.19 \Rightarrow (C)$$

2.

MFE, Spring 2009.

19. Consider a one-year 45-strike European put option on a stock S. You are given:

$T=1 \quad K=45$

(i) The current stock price, $S(0)$, is 50.00.

(ii) The only dividend is 5.00 to be paid in nine months. $D=5, \ t_D = 3/4$

(iii) $\mathrm{Var}[\ln F_{t,1}^{P}(S)] = \underbrace{0.01}_{\sigma^2} \times t, \quad 0 \le t \le 1.$ $\Rightarrow \sigma = 0.1$

(iv) The continuously compounded risk-free interest rate is 12%. $r=0.12$

Under the Black-Scholes framework, calculate the price of 100 units of the put option.

(A) 1.87

(B) 18.39

(C) 18.69

(D) 19.41

(E) 23.76

$$F_{0,1}^{P}(S) = 50 - 5e^{-3/4(0.12)} = 45.43 \quad \text{STORE}$$

$$d_1 = \frac{1}{0.1\sqrt{1}}\left[\ln\left(\frac{45.43}{45}\right) + \left(0.12 + \frac{0.01}{2}\right)\cdot 1\right]$$

$$\Rightarrow d_1 = 10\left[\ln\left(\frac{45.43}{45}\right) + 0.125\right]$$

$$d_1 = 1.345 \approx 1.35$$

$$\Rightarrow d_2 = d_1 - \sigma\sqrt{T} = 1.35 - 0.1 = 1.25.$$

$$N(-d_1) = 1 - N(1.35) = 1 - 0.9115 = 0.0885.$$

$$N(-d_2) = 1 - N(1.25) = 1 - 0.8944 = 0.1056$$

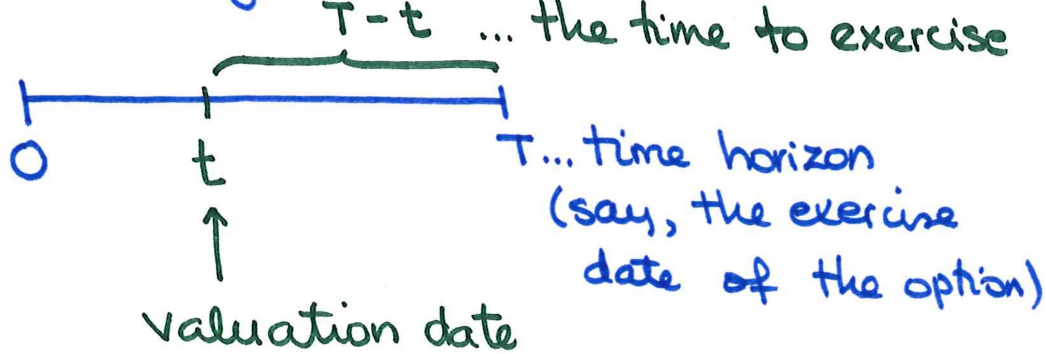
$$V_P(0) = 45e^{-0.12}(0.1056) - 45.43(0.0885)$$

$$= 0.1941 \Rightarrow (D)$$

3.

Option Greeks.

SET UP: Want to study the value of a portfolio as it depends on a set of independent arguments.



The underlying asset's price will be modeled in the Black-Scholes framework.

Recall our notation:

$\{S(t), t \geq 0\}$ ... this is a stock price process

- for any t, we know that $S(t)$ is a random variable; in particular, in the B·S model, it is LOGNORMAL, so that it can take any positive value

$\Rightarrow$ We introduce:

s... independent argument which stands for the CURRENT asset price

⇒ When we look at the pair (s, t), we are considering the

valuation time $\cdot\, t$

&

the the stock price @ that time is s.

Note: We have seen from our pricing formulae, that our prices also depend on:

$$r, \delta, \sigma$$

$$(K \ldots \text{strike price})$$

⇒ For our portfolios, their values @ time $\cdot\, t$ can be written as a function of five variables, i.e.,

$$v(s, t, r, \delta, \sigma)$$

$\uparrow$

value function

Def'n.

$$\frac{\partial}{\partial s} v(\ldots) =: \Delta(\ldots) \quad \text{DELTA}$$

$$\frac{\partial^2}{\partial s^2} v(\ldots) =: \Gamma(\ldots) \quad \text{GAMMA}$$

$$\frac{\partial}{\partial t} v(\ldots) =: \Theta(\ldots) \quad \text{THETA}$$

$$\frac{\partial}{\partial r} v(\ldots) =: \rho(\ldots) \quad \text{RHO}$$

$$\frac{\partial}{\partial \delta} v(\ldots) =: \Psi(\ldots) \quad \text{PSI}$$

$$\frac{\partial}{\partial \sigma} v(\ldots) =: \text{vega}(\ldots) \quad \text{VEGA}$$