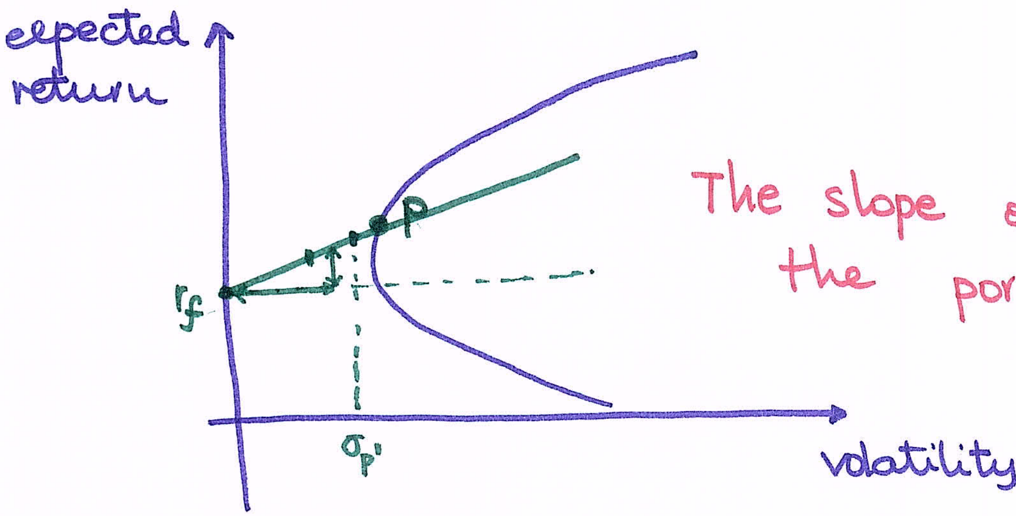
05/02/2018

Sharpe Ratio



The slope of this line is the portfolio P's SHARPE RATIO.

$$\frac{\mathbb{E}[R_P] - r_f}{\sigma_P} = \frac{\text{P's excess return}}{\text{P's volatility}}$$

Interpretation: Ratio of reward-to-risk.

8) You are given the following information about a two-asset portfolio:

(i) The Sharpe ratio of the portfolio is 0.3667. P... our portfolio

(ii) The risk-free rate is 4%. $r_f = 0.04$

(iii) If the portfolio were 50% invested in a risk-free asset and 50% invested in a risky asset X, its expected return would be 9.50%.

Now, assume that the weights were revised so that the portfolio were 20% invested in a risk-free asset and 80% invested in risky asset X. } P'

$\boxed{\sigma_{P'} = ?}$

Calculate the standard deviation of the portfolio return with the revised weights, assuming the Sharpe ratio remains the same.

(A) 6.0%

(B) 6.2%

(C) 12.8%

(D) 15.0%

(E) 24.0%

$$R_{P'} = 0.2 r_f + 0.8 \cdot R_X$$

Sharpe ratio of P':

$$\frac{\mathbb{E}[R_{P'}] - r_f}{\sigma_{P'}} = 0.3667$$

$$\frac{\mathbb{E}[0.2 r_f + \overbrace{0.8}^{x} \cdot R_X] - r_f}{\sigma_{P'}} = 0.3667$$

$$\frac{0.8(\overbrace{\mathbb{E}[R_X] - r_f}^{?})}{\underbrace{\sigma_{P'}}_{0.8 \cdot \sigma_X}} = 0.3667$$

The exp. return of portfolio P is 0.095 : $\left\{ \begin{array}{l} \frac{1}{2} \text{ weight given to X} \\ \frac{1}{2} \text{ weight in the risk-free asset} \end{array} \right.$

$$\mathbb{E}[R_P] = \frac{1}{2}\mathbb{E}[R_X] + \frac{1}{2} r_f = 0.095 \quad /(-r_f)$$

2.

$$\frac{1}{2}(\mathbb{E}[R_X] - r_f) = 0.095 - r_f = 0.095 - 0.04$$
$$= 0.055$$

$$\mathbb{E}[R_X] - r_f = 0.11$$

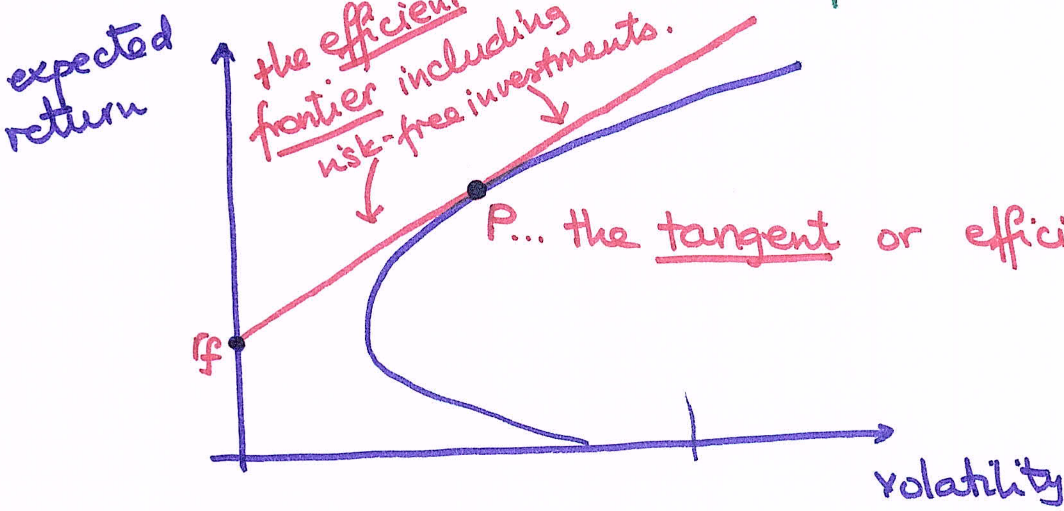
<u>We know:</u> $$\frac{\mathbb{E}[R_X] - r_f}{\sigma_X} = 0.3667$$

and $$\sigma_X = \frac{0.11}{0.3667}$$

We need: $$\sigma_{p'} = 0.8 \cdot \sigma_X = 0.8 \cdot \frac{0.11}{0.3667} \cong 0.24$$

Tangent Portfolio

The optimal portfolio to combine w/ the risk-free asset is the one w/ the highest Sharpe ratio, and still in the feasible set. So: it will end up on the efficient frontier!



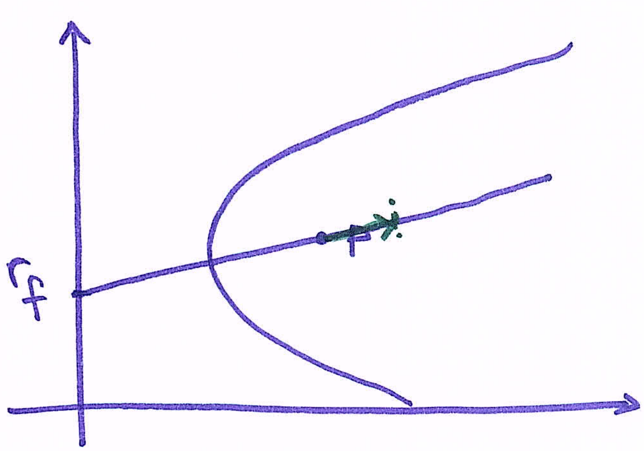
Section 11.6. Required Returns

Goal: To find out if we can improve a portfolio by adding (more) of a certain security.

Q: What is the condition for the improvement to happen, i.e, what is the new investments REQUIRED RETURN?

- Start w/ an arbitrary portfolio P.

Consider an investment I.



the Sharpe ratio of P is

$$\eta_P = \frac{\mathbb{E}[R_P] - r_f}{\sigma_P}$$

Let P' be obtained from P

by borrowing x (Value of P)

and

investing the proceeds in the investment I.

Assume the weight x is small!

- The new return: $R_{P'} = R_P - x \cdot r_f + x \cdot R_I$
- The new excess return:

$$\mathbb{E}[R_{P'}] - r_f = \underbrace{\mathbb{E}[R_P] - r_f}_{\text{the excess return of the old portfolio}} + x \underbrace{(\mathbb{E}[R_I] - r_f)}_{\text{the excess return of investment I}}$$

- The new portfolio's SD is ?

$$Var[R_{P'}] = Var[R_P - \underbrace{x \cdot r_f}_{\text{constant}} + x \cdot R_I]$$

$$= Var[R_P + x \cdot R_I]$$

$$= Var[R_P] + \underbrace{2 \cdot x \cdot Cov[R_P, R_I]}_{} + \underbrace{x^2 \cdot Var[R_I]}_{\text{to be diversified by } x \text{ small}}$$

$$f(y) = y^{1/2}$$

$$f'(y) = \frac{1}{2} \cdot \frac{1}{\sqrt{y}}$$

$$f(y_0 + dy) = f(y_0) + \frac{1}{2} \cdot \frac{1}{\sqrt{y_0}} dy + \text{lower order terms}$$

$$\sqrt{Var[R_{p'}]} = \sqrt{Var[R_p]} + \cancel{\frac{1}{2}} \cdot \frac{1}{\sqrt{Var[R_p]}} \cdot \cancel{2} \cdot x \, Cov[R_p, R_I]$$

$$SD[R_{p'}] = SD[R_p] + x \cdot \frac{\cancel{SD[R_p]} \cdot SD[R_I] \cdot corr[R_p, R_I]}{\cancel{SD[R_p]}}$$

$$= SD[R_p] + \underbrace{x \cdot SD[R_I] \cdot corr[R_p, R_I]}_{\text{"incremental" risk from adding investment } I}$$

Putting everything together, in order to improve the portfolio:

$$\cancel{x}(\mathbb{E}[R_I] - r_f) > \underbrace{\cancel{x} \cdot SD[R_I] \cdot corr[R_p, R_I] \cdot \overbrace{\eta_p}^{\text{sharpe ratio of portfolio } P}}_{\text{the effect of staying on the line w/ the same increase in risk}}$$

$$\Rightarrow \overbrace{\mathbb{E}[R_I] - r_f} > SD[R_I] \cdot corr[R_p, R_I] \cdot \frac{\overbrace{\mathbb{E}[R_p] - r_f}}{SD[R_p]} =: \beta_I^P$$

i.e.,

$$\mathbb{E}[R_I] > r_f + \beta_I^P (\mathbb{E}[R_p] - r_f)$$

<u>by def'n</u>: the beta of the investment I w/ portfolio P

<u>def'n.</u> $r_I \equiv r_f + \beta_I^P (\mathbb{E}[R_p] - r_f)$ ... the required return of investment I given portfolio P.

A portfolio is Efficient iff the EXECTED RETURN of every available security equals its required return (w/ respect to that portfolio):

⇒ If we have an efficient portfolio, for any security I its expected return:

$$\mathbb{E}[R_I] = r_I \equiv r_f + \beta_I^{eff} \left(\mathbb{E}[R_{eff}] - r_f \right)$$

β of investment I w/ the efficient portfolio

the return of the efficient portfolio