

Notes: This is a closed book and closed notes exam. The maximum number of points is 50.

Time: 50 minutes

Problem 2.1. (5 points) Assume the Black-Scholes framework for the evolution of a stock price. The stock pays no dividends. Consider a one-year European call on this stock. You are given the following:

- the call's delta is 0.6591,
- under the risk-neutral probability measure the probability that the option is in the money at expiration is 0.3409.

What is the volatility of this call option?

- (a) 0.3409
- (b) 1.6985
- (c) 2.0713
- (d) 3.0257
- (e) None of the above.

Problem 2.2. (5 points) Assume the Black-Scholes framework. For an at-the-money, T -year European call option on a non-dividend-paying stock you are given that its delta equals 0.5832. What is the delta of an otherwise identical option with exercise date at time $2T$?

- (a) 0.62
- (b) 0.66
- (c) 0.70
- (d) 0.74
- (e) None of the above.

Problem 2.3. (5 points) The current price of a non-dividend-paying stock is \$25 per share. A market-maker writes a three-month European put option on this stock and proceeds to delta-hedge it. The put premium is \$2.50, its delta is -0.30 , its gamma is 0.04 , and its theta is -0.01 per day.

The continuously compounded risk-free interest rate is 0.04 .

Assuming that the stock price does not change, what is the **approximate** overnight profit for the market-maker?

- (a) 0.011096
- (b) 0.018026
- (c) 0.021064
- (d) -0.014062
- (e) None of the above.

Problem 2.4. A market-maker sells option I for \$10. This option's delta is 0.6557 and its gamma is 0.02 . The market maker proceeds to delta-gamma hedge this commitment by trading in the underlying and also in option II on the same stock. The latter option's price is \$4.70, its delta is 0.5794 and its gamma is 0.04 .

What is the market-maker's resulting position in option II ?

- (a) Buy 0.5 of option II .
- (b) Write 0.5 of option II .
- (c) Buy 2 of option II .
- (d) Write 2 of option II .
- (e) None of the above.

Problem 2.5. (5 points) Assume the Black-Scholes model. Bertie Wooster was looking at stock-price and option data from yesterday. He decides to pose his friend Tuppy Glossop a riddle. Bertie tells Tuppy the following about yesterday's price of a stock and information on an option on this stock:

- the stock price yesterday was greater than \$77;
- the option's price was \$2.45;
- the option's delta was -0.1814 ;
- the option's gamma was 0.04 ;
- the option's theta was 0.01 **per day**.

Tuppy is allowed to see today's stock price and today's option price. They turn out to be \$80 and \$2.20, respectively. What is Tuppy going to guess to be yesterday's stock price?

- (a) \$77.08
- (b) \$77.27
- (c) \$78.63
- (d) \$78.22
- (e) None of the above.

Problem 2.6. Assume the Black-Scholes model. Let the current price of a continuous-dividend-paying stock be equal to \$80. Its dividend yield is 0.02 and its volatility is 0.20.

The continuously compounded risk-free interest rate is equal to 0.02.

Consider a one-year, at-the-money European call option on the above stock. What is the elasticity of that call option at time=0?

- (a) 0.15
- (b) 1.23
- (c) 4.56
- (d) 6.78
- (e) None of the above.

Problem 2.7. Assume the Black-Scholes model. The current price of a particular stock is \$100 per share.

Here is some information about the current prices and Greeks for a pair of European call options on this stock:

Strike price	80	90
Price	4.32	2.15
Δ	0.34	0.24

What is the current elasticity of the (80, 90)–bull spread constructed using the above options?

- (a) 4.61
- (b) 5.28
- (c) 6.34
- (d) 7.16
- (e) None of the above.

Problem 2.8. (5 points) Which of the following statements is **FALSE**?

- (a) The call theta is negative.
- (b) The put gamma is positive.
- (c) The vegas of otherwise identical calls and puts are equal.
- (d) The put rho is negative.
- (e) None of the above.

Problem 2.9. (5 points) Assume the Black-Scholes model. Let the current price of a non-dividend-paying stock be equal to \$80 per share. Its volatility is 0.20.

The continuously compounded risk-free interest rate is 0.04.

Consider a one-year, at-the-money European call option on the above stock. The current delta of the call option is 0.6179. What is the current gamma of the call option?

- (a) 0
- (b) 0.01256
- (c) 0.0238
- (d) 0.03862
- (e) None of the above.

Problem 2.10. Assume the Black-Scholes model. Let the current price of a continuous-dividend-paying stock be \$80. Its dividend yield is 0.03.

Let the continuously compounded risk-free interest rate be 0.05.

Consider a European call on the above stock with a quarter-year to exercise and with the strike price equal to $\$80e^{0.01}$. The current delta of this call option is 0.496264. What is the volatility of the stock?

- (a) 0.15
- (b) 0.2
- (c) 0.25
- (d) 0.3
- (e) None of the above.

Problem 2.11. Assume the Black-Scholes model. The current price of a continuous-dividend-paying stock is \$100. Its volatility is 0.25.

The continuously compounded risk-free interest rate is 0.04.

Consider an investor who buys a share of the above stock. Now, they want to create a Δ -neutral portfolio by writing European call options on this stock. You are given the following information about the call options:

- the time to exercise is T ,
- in our usual notation, $\delta T = 0.02$,
- in our usual notation, $d_1(S(0), 0) = 0.25$.

What is the number of call options that need to be written at time-0 to create a Δ -neutral portfolio?

- (a) 11.9043
- (b) 13.2603
- (c) 15.0457
- (d) 17.0403
- (e) None of the above.

Problem 2.12. Assume the Black-Scholes model. Let the current price of a non-dividend-paying stock be \$50. The stock's volatility is 0.25.

The continuously compounded risk-free interest rate is 0.03.

An investor writes a European call option with the following properties:

- the option is at the money at time-0,
- the option's time to exercise is four months,
- the option's premium is \$3.03,
- the option's delta is 0.5557.

The investor then, delta-hedges the written call by buying the underlying stock. The delta-hedge is not rebalanced for a month.

After one month, the investor realizes that the option is again at-the-money, and decides to liquidate the portfolio. What is their **exact** profit?

- (a) -0.07
- (b) 0.39
- (c) 1.24
- (d) 3.57
- (e) None of the above.