Multifactor Models [cont'd].

W: May 10th, 2019.

F1, F2 ... two well-diversified portfolios such that

$w_1 \cdot F1 + w_2 \cdot F2$ is an efficient portfolio

Consider a security S.

⋮

(LR) $R_S - r_f = \alpha_S + \beta_S^{F1}(R_{F1} - r_f) + \beta_S^{F2}(R_{F2} - r_f) + \varepsilon_S$

ε_S ~ noise/error term independent from R_{F1} & R_{F2}

We built a portfolio P:

$$R_P = r_f + \alpha_S + \varepsilon_S$$

Recall: ε_S is uncorrelated w/ R_{F1} & R_{F2}

$\Rightarrow Cov(R_{eff}, \varepsilon_S) = 0$

$\Rightarrow \varepsilon_S$ contains just diversifiable risk

$\Rightarrow$ Risk-premium of portfolio P is zero

$\Rightarrow \boxed{\alpha_S = 0}$

Take the expected value in the linear regression (LR) to get:

$$\mathbb{E}[R_S] = r_f + \beta_S^{F1}(\mathbb{E}[R_{F1}] - r_f) + \beta_S^{F2}(\mathbb{E}[R_{F2}] - r_f)$$

18) You are given the following information about the return of a security, using a two-factor model.

Factors	Beta	Expected Return
T	0.10	25 %
U	0.15	20 %

The annual effective risk-free rate of return is 5%.

Calculate the expected return of this security using the given two–factor model.

(A) 6.52%

(B) 8.33%

(C) 9.25%

(D) 11.33%

(E) 13.32%

$$E[R_s] = r_f + \beta_s^T (E[R_T] - r_f) + \beta_s^U (E[R_U] - r_f)$$

$$= 0.05 + 0.10(0.25 - 0.05) + 0.15(0.20 - 0.05)$$

$$= 0.05 + 0.02 + 0.0225 = 0.0925$$

We can generalize the model to multiple factors:

$$\mathbb{E}[R_S] = r_f + \sum_{i=1}^{N} \beta_S^{Fi} \left(\mathbb{E}[R_{Fi}] - r_f\right)$$

If all factor portfolios are self-financing:

$$\mathbb{E}[R_S] = r_f + \sum_{i=1}^{N} \beta_S^{Fi} \cdot \mathbb{E}[R_{Fi}]$$

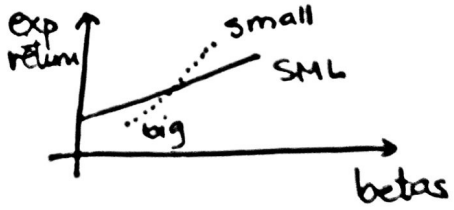
Factor Selection in a Multifactor Model
(aka Arbitrage Pricing Theory)

* Include: The Market Portfolio financed by the risk-free asset

Q: What other candidates do we have?

↙ Market Capitalization ↘ Book-to-Market ↘ Momentum

→ Market Capitalization



- Order the firms according to their MV.
- Find the median.
- Create S... an EQUALLY weighted portfolio w/ stocks below the median
- Create B... an EQUALLY weighted portfolio w/ stocks above the median
- Buy S and short B ⇒ get Small-minus-Big (SMB)

→ Book-to-Market Ratio.

- Order the firms according to their $\frac{BV}{MV}$.
- Construct L... an equally weighted portfolio of the lowest 30%
- Construct H... an equally weighted portfolio of the highest 70%
- Long H and short L (creating, again, a self-financing portfolio) ⇒ get the High-minus-Low Portfolio (HML).

→ Momentum Strategy.

- Order the stocks by their returns over the last year.
- Create a portfolio: { · LONG the top 30% · SHORT the bottom 30% }

so that the resulting portfolio is self-financing. You get the Prior-1-Year-Momentum PORTFOLIO (PR1YR)

⇒ Fama-French-Carhart Factor Specification:

$$\mathbb{E}[R_s] - r_f = \beta_s^{Mkt}\left(\mathbb{E}[R_{Mkt}] - r_f\right) + \beta_s^{SMB}\,\mathbb{E}[R_{SMB}] + \beta_s^{HML}\,\mathbb{E}[R_{HML}] + \beta_s^{PR1YR}\,\mathbb{E}[R_{PR1YR}]$$