

UNIVERSITY OF TEXAS AT AUSTIN

Log-normal stock prices: Tail probabilities.**Problem 4.1.** (15 points)

You are considering an investment in a non-dividend-paying stock versus an investment in a savings account. According to your belief, the stock's mean rate of return is α and its volatility is σ .

The continuously compounded interest rate is equal to r .

What is the probability that the stock outperforms the savings account at time $-T$? You should leave your final answer in terms of the function N .

Solution: With $Z \sim N(0, 1)$, we are looking for the probability

$$\begin{aligned} \mathbb{P}[S(T) > S(0)e^{rT}] &= \mathbb{P}\left[S(0)e^{(\alpha - \sigma^2/2)T + \sigma\sqrt{T}Z} > S(0)e^{rT}\right] \\ &= \mathbb{P}[(\alpha - \sigma^2/2)T + \sigma\sqrt{T}Z > rT] \\ &= \mathbb{P}\left[Z > \frac{\sqrt{T}}{\sigma}(r - \alpha + \sigma^2/2)\right] \\ &= N\left(\frac{(\alpha - r - \sigma^2/2)\sqrt{T}}{\sigma}\right). \end{aligned}$$

Problem 4.2. Assume the Black-Scholes framework. You are given the following information for a stock that pays dividends continuously at a rate proportional to its price:

- (i) The current stock price is \$250.
- (ii) The stock's volatility is 0.3.
- (iii) The continuously compounded expected rate of stock-price appreciation is 15%.

Find the value s^* such that

$$\mathbb{P}[S(4) > s^*] = 0.05.$$

- (a) \$861.65
- (b) \$874.18
- (c) \$889.94
- (d) \$905.48
- (e) None of the above.

Solution: (e)

$$s^* = 250e^{(0.15 - 0.045)(4) + 0.3(2)(1.645)} = 1020.92$$