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UNIVERSITY OF TEXAS AT AUSTINQuiz #1

Prerequisite material.

**Problem 1.1.** (2 pts) If  $X$  and  $Y$  are independent random variables, then

$$F_{X+Y}(a) = F_X(a) \cdot F_Y(a).$$

*True or false?***Problem 1.2.** (2 points) Let  $X$  be a normal random variable with parameters  $(\mu = 2, \sigma^2 = 1)$ , and let  $Y$  be a normal random variable with parameters  $(\mu = -2, \sigma^2 = 1)$ . Assume that  $X$  and  $Y$  are independent. Then, the variance of the random variable  $X + Y$  equals 2. *True or false?***Problem 1.3.** (2 points) In our usual notation, let  $S(0) = 40, r = 0.08, \sigma = 0.3, \delta = 0$ . You need to construct a 2-period forward binomial tree for the above stock with every period in the tree of length  $h = 0.5$ . Then,  $u > 1.45$ . *True or false?***Problem 1.4.** (2 points) You are using a binomial asset-pricing model to model the evolution of the price of a particular stock. Then, the  $\Delta$  in the replicating portfolio of a single call option on that stock never exceeds 1. *True or false?***Problem 1.5.** (8 points)Let  $X$  be a continuous random variable with probability density function  $f_X(x)$ . Let its cumulative distribution function be denoted by  $F_X(x) = \mathbb{P}[X \leq x]$ . Define the new random variable  $Y$  as

$$Y = F_X(X).$$

Find  $\mathbb{E}[Y]$ .