Problem 2.1. (5 pts) Assume the Black-Scholes framework for modeling the futures prices on oil. Let the futures price of oil for delivery in one year equal $90.00. The volatility of this price is given to be 0.26. Assume that the continuously compounded risk-free interest rate equals 0.05. Consider an at-the-money European put option on the above futures contract with the exercise date in one year.

What is the Black-Scholes price $V_P(0)$ of this put option?

(a) $6.62$
(b) $7.73$
(c) $8.85$
(d) $9.31$
(e) None of the above.

Solution: (c)

We are given the one-year futures price on oil to be $F_{0,1} = 90$.

Since the option in question is given to be at-the-money, it is immediate that in our usual notation,

$$d_1 = \frac{1}{2}\sigma\sqrt{T} = \frac{0.25}{2} = 0.13, \quad d_2 = -d_1 = -0.13.$$

The price of the put option is

$$V_P(0) = F_{0,1}e^{-rT}(2N(-d_1) - 1) = 90e^{-0.05(2 \times 0.5517 - 1)} = 8.85.$$

Problem 2.2. Assume the Black-Scholes framework. For an at-the-money, $T$-year European call option on a non-dividend-paying stock you are given that its delta equals 0.5832. What is the delta of an otherwise identical option with exercise date at time $2T$?

(a) 0.62
(b) 0.66
(c) 0.70
(d) 0.74
(e) None of the above.

Solution: (a)

Problem 2.3. Which of the following greeks is usually negative?

(a) Call delta.
(b) Call gamma.
(c) Call theta.
(d) Call vega.
(e) None of the above.

Solution: (c)