M339W/389W Financial Mathematics for Actuarial Applications
University of Texas at Austin
Some Sample Problems for In-Term Exam I
Instructor: Milica Čudina

Notes: This is a closed book and closed notes exam.
Time: 50 minutes

TRUE/FALSE

1 (2)	TRUE	FALSE
2 (2)	TRUE	FALSE
3 (2)	TRUE	FALSE
4 (2)	TRUE	FALSE
5 (2)	TRUE	FALSE

MULTIPLE CHOICE

1 (5)	a	b	c	d	e
2 (5)	a	b	c	d	e
3 (5)	a	b	c	d	e
4 (5)	a	b	c	d	e
5 (5)	a	b	c	d	e
6 (5)	a	b	c	d	e

FOR GRADER'S USE ONLY:

T/F	1.	2.	M.C.	Σ

1.1. **TRUE/FALSE QUESTIONS.** *Please note your answers on the front page.*

Problem 1.1. (2 points) Under the risk-neutral probability measure, every option on a particular stock has the continuously compounded, risk-free interest rate as its mean rate of return. *True or false?*

Problem 1.2. The Black-Scholes option pricing formula can **always** be used for pricing American-type call options on non-dividend-paying assets. *True or false?*

Problem 1.3. Continuously compounded returns of stocks are multiplicative.

Problem 1.4. Let X be a strictly positive random variable, then

$$\mathbb{E}[X] = |\mathbb{E}[X]|.$$

Problem 1.5. In the lognormal stock-price model, the mean time$-t$ stock price is increasing as a function of t.

Problem 1.6. Let the stock price be modeled by a lognormal distribution. Then, the median stock price always exceeds the mean stockprice. *True or false?*

Problem 1.7. Let the stock price be modeled by a lognormal distribution. Then, the expected payoff of a European put option with exercise date T and strike K greater than or equal to $\max(0, K - \mathbb{E}[S(T)])$. *True or false?*

Problem 1.8. In developing the log-normal stock price model, we assumed independent, identically distributed realized returns.

True or false?

Problem 1.9. (2 pts)

In the setting of the Black-Scholes stock-price model, let $\mathbf{S} = \{S(t), t \geq 0\}$ denote the stock price whose drift is α and volatility is σ.

Then,

$$\mathbb{E}[S(t+h) \mid S(t)] = S(t), \text{ for every } t \geq 0 \text{ and } h > 0.$$

Problem 1.10. The product of log-normal random variables is normal.

1.2. **FREE-RESPONSE PROBLEMS.** Please, explain carefully all your statements and assumptions. Numerical results or single-word answers without an explanation (even if they're correct) are worth 0 points.

Problem 1.11. (15 points)

Consider a non-dividend-paying stock whose price is modeled using the lognormal distribution. Suppose that the current stock price equals \$100 and that its volatility is given to be 0.2.

The continuously compounded, risk-free interest rate is assumed to equal 0.04.

Consider a derivative security which entitles its owner to obtain a European call option on the above stock six months from today, i.e., at time $t_* = 1/2$. The call option is to be half a year to expiration at time of delivery and have the strike equal to 105% of the time$-t^*$ price of the underlying asset. This contract is called a **forward start option**.

What is the price of the forward start option?

Problem 1.12. (10 points) Let the stock prices be modeled using the lognormal distribution. The mean stock price at time-1 equals 120 and the median stock price 115. What is the probability that the time-1 stock price exceeds 100?

1.3. **MULTIPLE CHOICE QUESTIONS.**

Problem 1.13. (5 points) The current price of a continuous-dividend-paying stock is given to be \$92. The stock's volatility is 0.35 and its dividend yield is 0.02.

The continuously compounded risk-free interest rate is 0.05.

Consider a \$90-strike European call option on the above stock with exercise date in a quarter-year.

What is the Black-Scholes price of this call option?

(a) 5.05
(b) 7.66
(c) 7.71
(d) 7.89
(e) None of the above.

Problem 1.14. A continuous-dividend-paying stock has a dividend yield of 0.02. The time$-t$ realized (rate of) return is modeled as

$$R(0,t) \sim N(\text{mean} = 0.03t, \text{variance} = 0.09t)$$

Find the probability that the time-4 stock price exceeds today's stock price.

(a) 0.4920
(b) 0.4960
(c) 0.5040
(d) 0.5080
(e) None of the above.

Problem 1.15. (5 pts) Assume the Black-Scholes framework. Let the current price of a non-dividend-paying stock be equal to $S(0) = 95$ and let its volatility be equal to 0.35. Consider a European call on that stock with strike 100 and exercise date in 9 months. Let the risk-free continuously compounded interest rate be 6% per annum.

Denote the price of the call by $V_C(0)$. Then,

(a) $V_C(0) < \$5.20$
(b) $\$5.20 \leq V_C(0) < \$7.69$
(c) $\$7.69 \leq V_C(0) < \$9.04$
(d) $9.04 \leq V_C(0) < \$11.25$
(e) None of the above.

Problem 1.16. (5 pts) Assume the Black-Scholes framework. Let the current price of a share of stock be equal to $S(0) = 80$, let its volatility be $\sigma = 0.3$, and let $\delta = 0$ (in our usual notation).

Consider a gap option with expiration date $T = 1$ year such that its payoff is $S(T) - 90$ if $S(T) > 100$.

You are given that the continuously compounded risk-free interest rate equals $r = 0.05$ per annum.

Let $V_{GC}(0)$ denote the price of the above gap option. Then,

(a) $V_{GC}(0) < \$3.20$
(b) $\$3.20 \leq V_{GC}(0) < \$5.69$
(c) $\$5.69 \leq V_{GC}(0) < \$7.04$
(d) $7.04 \leq V_{GC}(0) < \$11.25$
(e) None of the above.

Problem 1.17. Assume the Black-Scholes setting. Let $S(0) = \$63.75, \sigma = 0.20, r = 0.055$. The stock pays no dividend and the option expires in 50 days (simplify the number of days in a year to 360).

What is the price of a \$60-strike European put?

(a) 0.66
(b) 0.55
(c) 0.44
(d) 0.37
(e) None of the above.

Problem 1.18. Assume the Black-Scholes setting.
Assume $S(0) = \$28.50, \sigma = 0.32, r = 0.04$. The stock pays a 1.0% continuous dividend and the option expires in 110 days (simplify the number of days in a year to 360).
What is the price of a \$30-strike put?

(a) 2.75
(b) 2.10
(c) 1.80

(d) 1.20
(e) None of the above.

Problem 1.19. (5 points) Consider a non-dividend-paying stock whose price $\mathbf{S} = \{S(t), t \geq 0\}$ is modeled using a geometric Brownian motion. Suppose that the current stock price equals \$100 and that its volatility is given to be 0.25.

The continuously compounded, risk-free interest rate is assumed to equal 0.04.

Consider a derivative security which entitles its owner to obtain a European call option on the above stock six months from today, i.e., at time $t_* = 1/2$. The call option is to be 3−month to expiration at time of delivery and have the strike equal to 105% of the time−t^* price of the underlying asset. This contract is called a **forward start option**.

What is the price of the forward start option?

(a) 3.15
(b) 8.65
(c) 10.51
(d) 13.55
(e) None of the above.

Problem 1.20. Assume the Black-Scholes setting.
Today's price of a non-dividend paying stock is \$65, and its volatility is 0.20.

The continuously compounded risk-free interest rate is 0.055.

What is the price of a three-month, \$60-strike European put option on the above stock?

(a) 0.66
(b) 0.59
(c) 0.44
(d) 0.37
(e) None of the above.

Problem 1.21. Assume the Black-Scholes setting.

Today's stock price is observed to be $S(0) = \$30$ per share. Its dividend yield is given to be 0.01 and its volatility equals 0.30.

The continuously compounded risk-free interest rate is $r = 0.04$.

What is the price of a half-year, \$30-strike put?

(a) 2.75
(b) 2.38
(c) 1.80
(d) 1.20
(e) None of the above.