

Name:

M339W/389W Financial Mathematics for Actuarial Applications

University of Texas at Austin

## The Prerequisite In-Term Exam

Instructor: Milica Čudina

**Notes:** This is a closed book and closed notes exam. The maximum number of points on this exam is 100.

**Time:** 50 minutes

## MULTIPLE CHOICE

1.14 (5)	a	b	c	d	e
1.15 (5)	a	b	c	d	e
?? (5)	a	b	c	d	e
?? (5)	a	b	c	d	e
?? (5)	a	b	c	d	e

FOR GRADER'S USE ONLY:

[illegible]

1.1. DEFINITION.**Problem 1.1.** (10 points)Provide the definition of an arbitrage portfolio.1.2. TRUE/FALSE QUESTIONS.**Problem 1.2.** (2 points) Consider two exchange options, one that allows you to exchange a share of asset  $S$  for a share of asset  $Q$ , and another one that allows you to forfeit a share of asset  $Q$  and obtain a share of asset  $S$  in return. Assume neither of the two stocks pays dividends.

Then, the current spot prices of the two assets are the same if and only if the two exchange options have the same price.

**Problem 1.3.** (2 pts) A non-dividend-paying stock sells today for \$100 per share. The yearly effective interest rate is 0.21. Then,  $F_{0,1/2}(S) > 110$ .**Problem 1.4.** (2 points) In the binomial asset pricing model, the replicating portfolio for a put option has a bond investment which is equivalent to borrowing at the risk-free interest rate. *True or false?***Problem 1.5.** (2 points) Gap call options are always worth more than otherwise identical call options with the same strike price. *True or false?***Problem 1.6.** (2 pts) If the random variable  $X$  is standard normal, then the distribution function of the random variable  $Y = |X|$  equals

$$F_Y(a) = 2\Phi(a) - 1 \text{ for every } a \geq 0.$$

*True or false?***Problem 1.7.** (2 points) It is never optimal to exercise an American call option on a non-dividend paying stock early. *True or false?*

1.3. **Free-response problems.** Please, explain carefully all your statements and assumptions. Numerical results or single-word answers without an explanation (even if they're correct) are worth 0 points.

**Problem 1.8.** (15 points)

Two scales are used to measure the mass  $m$  of a precious stone. The first scale makes an error in measurement which we model by a normally distributed random variable with mean  $\mu_1 = 0$  and standard deviation  $\sigma_1 = 0.04m$ . The second scale is more accurate. We model its error by a normal random variable with mean  $\mu_2 = 0$  and standard deviation  $\sigma_2 = 0.03m$ .

We assume that the measurements made using the two different scales are independent.

To get our final estimate of the mass of the stone, we take the average of the two results from the two different scales.

What is the probability that the value we get is within  $0.005m$  of the actual mass of the stone?

**Problem 1.9.** (10 points) Assume that  $Y_1 = e^X$  where  $X$  is a standard normal random variable.

(i) (2 points) What is the probability that  $Y_1$  exceeds 5?

(ii) (3 + 5 points) Find the mean and the variance of  $Y_1$ .

*Hint:* It helps if you use the expression for the moment generating function of a standard normal random variable.

**Problem 1.10.** (5 points) *Source: FM(DM) sample problem #42.*

An investor purchases one share of a non-dividend-paying stock and writes an at-the-money,  $T$ -year, European call option in this stock. The call premium is denoted by  $C$ . Assume that there are no transaction costs. The continuously compounded, risk-free interest rate is denoted by  $r$ . Let the argument  $s$  represent the stock price at time  $T$ .

- (i) (3 points) Determine an algebraic expression for the investor's profit at expiration  $T$  in terms of  $C, r, T$  and the strike  $K$ .
- (ii) (2 points) In particular, how does the expression you obtained in (i) simplify if the call is in-the-money on the exercise date?

**Problem 1.11.** (15 points) The random vector  $(X_1, X_2, X_3)$  is jointly normal. Its marginal distributions are:

$$X_1 \sim N(\text{mean} = 0, \text{variance} = 4), X_2 \sim N(\text{mean} = 1, \text{variance} = 1), X_3 \sim N(\text{mean} = -1, \text{variance} = 9).$$

The correlation coefficients are given to be

$$\text{corr}[X_1, X_2] = 0.3, \text{corr}[X_2, X_3] = 0.4, \text{corr}[X_1, X_3] = -0.3.$$

What is the distribution of the random variable  $X = X_1 - X_2 + 2X_3$ ? Please, provide the **name** of the distribution, as well as the **values** of its parameters.

**Problem 1.12.** (10 pts) For a two-period binomial model, you are given that:

- (1) each period is one year;
- (2) the current price of a non-dividend-paying stock  $S$  is  $S(0) = \$20$ ;
- (3)  $u = 1.3$ , with  $u$  as in the standard notation for the binomial model;
- (4)  $d = 0.9$ , with  $d$  as in the standard notation for the binomial model;
- (5) the continuously compounded risk-free interest rate is  $r = 0.05$ .

Consider a special call option which pays the excess above the strike price  $K = 23$  (if any!) at the end of every binomial period.

Find the price of this option.

**Problem 1.13.** (5 points)

Emmanuel entered an extra special kind of game with his friend Fischer. First, they toss a fair coin. If the coin comes up heads, Emmanuel gives \$5,000 to Fischer. If the coin comes up tails, Fischer gives \$2,000 to Emmanuel. Then, regardless of the outcome of the first cointoss, they toss the same fair coin again. If it comes up heads, Emmanuel gives Fischer \$4,000. If the coin comes up tails, Fischer gives \$3,000 to Emmanuel. What is the expected cashflow, i.e., what is the expected amount of money that changes hands and who gives it to whom?

1.4. **MULTIPLE CHOICE QUESTIONS.** *Please note your answers on the front page.*

**Problem 1.14.** (5 pts) Consider a continuous-dividend-paying stock currently priced at \$100 per share whose dividend yield is projected to equal 0.02.

The price of this stock in one year is modeled using a one-period binomial tree. The down factor  $d$  is given to be 0.83, while the up factor  $u$  is unknown. (If you want to you can construct a cutesy narrative about coffee being spilled on the model write-up or some such.)

The continuously compounded risk-free interest rate is given to equal 0.04.

You observe that the price of a one-year, \$110-strike European call option on this stock consistent with the above model equals \$5.15. Find the price of the otherwise identical put option.

- (a) \$10.84
- (b) \$12.82
- (c) \$15.15
- (d) \$17.13
- (e) None of the above.

**Problem 1.15.** (5 pts) Consider a non-dividend-paying stock currently priced at \$100 per share.

The price of this stock in one year is modeled using a one-period binomial tree under the assumption that the stock price can either go up to 110 or down to 90.

Let the continuously compounded risk-free interest rate equal 0.04. What is the risk-neutral probability of the stock price going up?

- (a) About 0.2969
- (b) About 0.3039
- (c) About 0.5000
- (d) About 0.7041
- (e) None of the above.



**Problem 1.16.** (5 points) The current stock price is 40 per share. The price at the end of a three-month period is modeled with a one-period binomial tree so that the stock price can either increase by \$10, or decrease by \$4. The stock pays dividends continuously with the dividend yield 0.04.

The continuously compounded risk-free interest rate is 0.05.

What is the stock investment in a replicating portfolio for three-month, \$40-strike European **straddle** on the above stock?

- (a) Long 0.42 shares
- (b) Long 0.71 shares
- (c) Short 0.71 shares
- (d) Short 0.42 shares
- (e) None of the above.