M339W/389W Financial Mathematics for Actuarial Applications
University of Texas at Austin
Sample In-Term Exam 2
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Notes: This is a closed book and closed notes exam.
Time: 50 minutes

TRUE/FALSE

1 (2)	TRUE	FALSE
2 (2)	TRUE	FALSE
3 (2)	TRUE	FALSE
4 (2)	TRUE	FALSE
5 (2)	TRUE	FALSE

MULTIPLE CHOICE

1 (5)	a	b	c	d	e
2 (5)	a	b	c	d	e
3 (5)	a	b	c	d	e
4 (5)	a	b	c	d	e
5 (5)	a	b	c	d	e
6 (5)	a	b	c	d	e

FOR GRADER'S USE ONLY:

T/F	1.	2.	M.C.	Σ

2.1. **TRUE/FALSE QUESTIONS.** *Please note your answers on the front page.*

Problem 2.1. Gamma of a call bull spread is always positive. *True or false?*

Problem 2.2. Assume the Black-Scholes model. The elasticity of a European put option is always nonpositive.

True or false?

Problem 2.3. (2 points) In order to **both** delta hedge and gamma hedge a position in a certain option, the market-maker must trade in another type of option (i.e., not only in the money-market and the underlying risky asset). *True or false?*

Problem 2.4. (2 points) Market makers usually do not need to rebalance their portfolios after the initial hedge is established. *True or false?*

Problem 2.5. (2 points) A market maker who delta-hedges **completely** insures himself against losses. *True or false?*

2.2. **FREE-RESPONSE PROBLEMS.** Please, explain carefully all your statements and assumptions. Numerical results or single-word answers without an explanation (even if they're correct) are worth 0 points.

Problem 2.6. (15 points) Consider a non-dividend-paying stock whose current price is \$45 per share. Its volatility is given to be 0.20.

The continuously compounded risk-free interest rate is 0.04.

A market maker sells a European, 91$-$day, \$50-strike call option on the above stock for \$0.42 and delta-hedges the commitment using shares of stock. The call's delta at time-0 is 0.1841. The market-maker does not update the delta-hedge for a week. Then, she realizes that the call option is at-the-money and decides to liquidate the entire portfolio. What is the market maker's profit?

Problem 2.7. (10 points) Let $S(t)$ denote the time$-t$ price of a continuous-dividend-paying stock with dividend yield δ and volatility σ.

The continuously compounded risk-free interest rate is denoted by r.

You write a special option which pays $\min(S(T), K)$ for a positive monetary amount K at time$-T$. You want to delta-hedge this commitment. What is the time-0 delta of the special option, expressed using the notation given above?

Problem 2.8. (10 points) Assume the Black-Scholes framework for a non-dividend-paying stock whose current price is \$51.

A market-maker writes a European call option and sells it for \$9.25. Then, the market-maker delta-hedges by trading in the shares of the underlying stock. You are given the following current values of the greeks of the call option:

- the Δ is 0.66;
- the Γ is 0.02;
- the Θ is -0.01 per day.

The continuously compounded risk-free interest rate is 0.04.

Using the delta-gamma-theta approximation, calculate the approximate profit for the market-maker after one day if the stock price drops to \$50.

2.3. MULTIPLE CHOICE QUESTIONS.

Problem 2.9. (5 points) Assume the Black-Scholes framework.

The goal is to delta-hedge a one-year, at-the-money straddle on a non-dividend-paying stock whose current price is \$50. The stock's volatility is 0.20.

The continuously compounded risk-free interest rate is 0.10.

What is the cost of delta-hedging the straddle using shares of the underlying stock?

(a) \$22.58
(b) \$23.23
(c) \$24.33
(d) \$25.19
(e) None of the above.

Problem 2.10. (5 points) Assume the Black-Scholes framework. The current stock price is \$50 per share. Its dividend yield is 0.01 and its volatility is 0.25.

The continuously compounded risk-free interest rate is 0.05.

Consider a one-year, \$55-strike European put option on the above stock. What is the volatility of the put option?

(a) 1.013
(b) -0.534
(c) 6.6
(d) 0.978
(e) None of the above.

Problem 2.11. Which of the following greeks is usually negative?

(a) Call delta.
(b) Call gamma.
(c) Call theta.
(d) Call vega.
(e) None of the above.

Problem 2.12. (5 points) *Source: Sample IFM (Derivatives:Advanced) Problem #8.*
Consider a non-dividend-paying stock whose price $\mathbf{S} = \{S(t), t \geq 0\}$ is modeled using the Black-Scholes model. Suppose that the current stock price equals \$40 and that its volatility is given to be 0.30.

Consider a three-month, \$41.5-strike European call option on the above stock. You learn that the current call delta equals 0.5.

What is the Black-Scholes price of this call option?

(a) 2.19
(b) 2.65
(c) 3.51
(d) 3.65
(e) None of the above.

Problem 2.13. Consider the following portfolio:

- 5 long options of type I,
- 4 long options of type II,
- 1 written option of type III.

The prices of the three options are 0.75, 1.00, and 1.50, respectively, while the option elasticities are 10, 7, and 2, respectively. What is the elasticity of the above portoflio?

(a) 5
(b) 7
(c) 10
(d) 12
(e) None of the above.

Problem 2.14. (5 points) Assume the Black-Scholes model is used. The current price of a continuous-dividend-paying stock is \$50. Its dividend yield is given to be 0.03.

The continuously compounded, risk-free interest rate equals 0.03.

You observe the price of an at-the-money, one-year European put option on the stock as equal to \$6.93. What is the implied volatility of the stock?

(a) 0.18
(b) 0.24
(c) 0.36
(d) 0.42
(e) None of the above.

Problem 2.15. (5 points) The current stock price is equal to \$50. Consider a European call option whose current price is \$3.43. The call's current Δ is 0.60 and its Γ is 0.02. What is the approximate call price if the stock price increases to \$52 in a short time interval?

(a) 4.03

(b) 4.27

(c) 4.41

(d) 4.67

(e) None of the above.