An introduction to European put options. Moneyness.

5.1. Put options. A put option gives the owner the right – but not the obligation – to sell the underlying asset at a predetermined price during a predetermined time period. The seller of a put option is obligated to buy if asked. The mechanics of the European put option are the following:

at time−0: (1) the contract is agreed upon between the buyer and the writer of the option,
    (2) the logistics of the contract are worked out (these include the underlying asset, the expiration date \( T \) and the strike price \( K \)),
    (3) the buyer of the option pays a premium to the writer;

at time−\( T \): the buyer of the option can choose whether he/she will sell the underlying asset for the strike price \( K \).

5.1.1. The European put option payoff. We already went through a similar procedure with European call options, so we will just briefly repeat the mental exercise and figure out the European-put-buyer’s profit.

If the strike price \( K \) exceeds the final asset price \( S(T) \), i.e., if \( S(T) < K \), this means that the put-option holder is able to sell the asset for a higher price than he/she would be able to in the market. In fact, in our perfectly liquid markets, he/she would be able to purchase the asset for \( S(T) \) and immediately sell it to the put writer for the strike price \( K \). The payoff is, thus, \( S(T) - K \).

To the contrary, if \( S(T) \geq K \), the put-option owner would be better of selling the asset at the market price. So, he/she will simply walk away from the contract incurring the payoff of 0.

Combining the above two states of the world, we get the following expression for the long-put-option payoff:

\[
V_P(T) = \max(K - S(T), 0) = (K - S(T))_+. 
\]

So, the payoff function for a put option is

\[
v_P(s) = (K - s)_+. 
\]

For \( K = 1000 \), we get the payoff curve below (in blue). The buyer is supposed to pay the premium at \( t = 0 \). This will affect the profit curve. For instance, if the initial premium for this option with exercise date in one year equals $50 and if the continuously compounded interest rate equals \( r = 0.06 \), then the profit curve is the one graphed below in red.
Looking at the graph above, we see that the put-option payoff (as well as profit) is decreasing in the asset price and bounded from above by the strike price $K$.

**Example 5.1. Put option on a market index**

Consider a put option on a market index with exercise date in six months and with strike price $K = 1000$. Assume that the premium for this option equals $V_P(0) = $80 and that the effective interest rate for the six-month period equals $i = 0.03$. The payoff function is

$$v_P(s) = (1000 - s)_+$$

and the profit function is

$$v_P(s) - V_P(0)(1 + i) = (1000 - s)_+ - 80 \cdot 1.03 = (1000 - s)_+ - 82.4.$$

Once the exercise date is reached, one gets to observe the final index value and calculate the realized payoff and profit. For instance:

1. If the final index value equals $S(T) = 1050$, the put-owner’s payoff is 0 (the option is not even exercised). The profit is, hence, $-82.40$. So, the owner of the option experiences a loss of $82.40$.

2. If the final index value equals $S(T) = 800$, the payoff is $1000 - 800 = 200$ (the option is, indeed, exercised). The profit is, hence, $200 - 82.40 = 117.60$. So, the owner of the option gains of $117.60$.

**Remark 5.2.** Two positions in the market with the the payoff of one being the exact negative payoff of the other are said to be opposites of each other. In particular,

- a purchased call is the opposite of a written call;
- a purchased put option is the opposite of a written put.

5.1.2. **Suggested problems.** McDonald: #2.3, #2.5,#2.14; Sample FM (Derivatives Markets): Problem #12.
5.2. **Moneyness.** The moneyness of an option reflects whether an option would cause a positive, negative or zero payoff were to be exercised immediately. More precisely, at any time \( t \in [0, T] \), an option is said to be:

1. **in-the-money** – if there is strictly positive payoff if the option is exercised immediately;
2. **at-the-money** – if there is zero payoff if exercised immediately;
3. **out-of-the-money** – if there is negative payoff if exercised immediately.

**Example 5.3. Moneyness of a put option**

Consider a put option with strike \( K = 100 \). If the initial price \( S(0) \) of the underlying asset equals:

1. 95 – then the option is in-the-money;
2. 100 – then the option is at-the-money;
3. 105 – then the option is out-of-the-money.

Imagine that we are half-way through the life of the option, i.e., we have reached time \( T/2 \). We can observe the price of the underlying asset at that time too. We denote this value by \( S(T/2) \), and we can also state that at time \(-T/2\)

1. if \( S(T/2) > 100 \) the put option is out-of-the-money;
2. if \( S(T/2) = 100 \) the put option is at-the-money;
3. if \( S(T/2) < 100 \) the put option is in-the-money.