UNIVERSITY OF TEXAS AT AUSTIN

Lecture 5

An introduction to European put options. Moneyness.

5.1. **Put options.** A *put option* gives the owner the **right** – but **not** the obligation – to sell the underlying asset at a predetermined price during a predetermined time period. The seller of a put option is **obligated** to buy if asked. The mechanics of the European put option are the following:

at time−0: (1) the contract is agreed upon between the buyer and the writer of the option,
(2) the logistics of the contract are worked out (these include the underlying asset, the *expiration date* T and the *strike price* K),
(3) the buyer of the option pays a *premium* to the writer;

at time−T: the buyer of the option can **choose** whether he/she will sell the underlying asset for the strike price K.

5.1.1. *The European put option payoff.* We already went through a similar procedure with European call options, so we will just briefly repeat the mental exercise and figure out the European-put-buyer's profit.

If the strike price K exceeds the final asset price $S(T)$, i.e., if $S(T) < K$, this means that the put-option holder is able to sell the asset for a higher price than he/she would be able to in the market. In fact, in our perfectly liquid markets, he/she would be able to purchase the asset for $S(T)$ and immediately sell it to the put writer for the strike price K. The payoff is, thus, $S(T) - K$.

To the contrary, if $S(T) \geq K$, the put-option owner would be better of selling the asset at the market price. So, he/she will simply walk away from the contract incurring the payoff of 0.

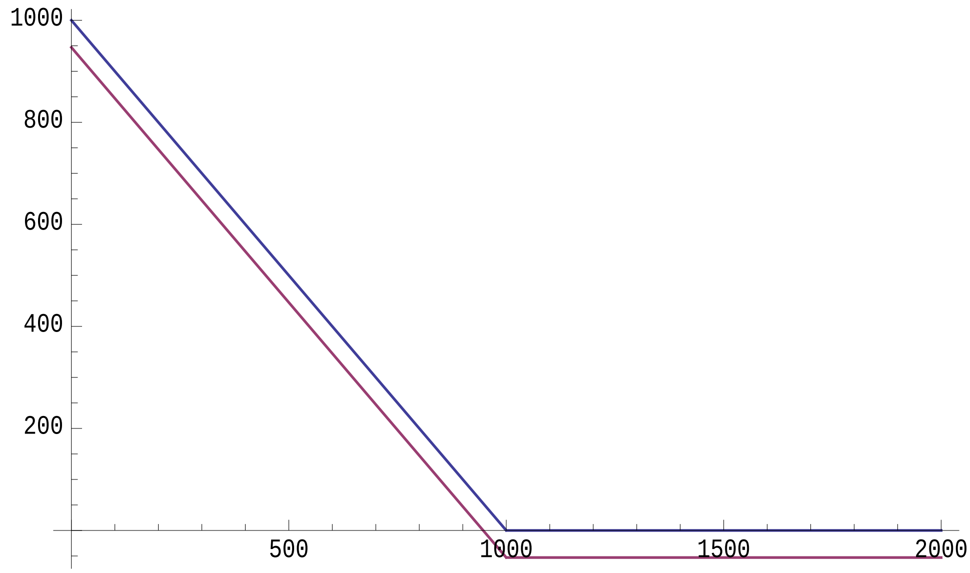
Combining the above two states of the world, we get the following expression for the long-put-option payoff:

$$V_P(T) = \max(K - S(T), 0) = (K - S(T))_+.$$

So, the payoff function for a put option is

$$v_P(s) = (K - s)_+.$$

For $K = 1000$, we get the payoff curve below (in blue). The buyer is supposed to pay the premium at $t = 0$. This will affect the profit curve. For instance, if the intial premium for this option with exercise date in one year equals \$50 and if the continusouly compounded interest rate equals $r = 0.06$, then the profit curve is the one graphed below in red.



Looking at the graph above, we see that the put-option payoff (as well as profit) is decreasing in the asset price and bounded from above by the strike price K.

Example 5.1. **Put option on a market index**
Consider a put option on a market index with exercise date in six months and with strike price $K = 1000$. Assume that the premium for this option equals $V_P(0) = \$80$ and that the effective interest rate for the six-month period equals $i = 0.03$. The payoff function is

$$v_P(s) = (1000 - s)_+$$

and the profit fuction is

$$v_P(s) - V_P(0)(1+i) = (1000 - s)_+ - 80 \cdot 1.03 = (1000 - s)_+ - 82.4.$$

Once the exercise date is reached, one gets to observe the final index value and calculate the realized payoff and profit. For instance:

(1) If the final index value equals $S(T) = 1,050$, the put-owner's payoff is 0 (the option is not even exercised). The profit is, hence, -82.40. So, the owner of the option experiences a **loss** of \$82.40.
(2) If the final index value equals $S(T) = 800$, the payoff is $1000 - 800 = 200$ (the option is, indeed, exercised). The profit is, hence, $200 - 82.40 = 117.60$. So, the owner of the option **gains** of \$117.60.

Remark 5.2. Two positions in the market with the the payoff of one being the exact negative payoff of the other are said to be **opposites** of each other. In particular,

- a **purchased** call is the opposite of a **written** call;
- a **purchased** put option is the opposite of a **written** put.

5.1.2. *Suggested problems.* McDonald: #2.3, #2.5,#2.14; Sample FM (Derivatives Markets): Problem #12.

5.2. **Moneyness.** The **moneyness** of an option reflects whether an option would cause a positive, negative or zero payoff were to be exercised **immediately**. More precisely, at any time $t \in [0, T]$, an option is said to be:

(1) *in-the-money* – if there is strictly **positive** payoff if the option is exercised immediately;
(2) *at-the-money* – if there is **zero** payoff if exercised immediately;
(3) *out-of-the money* – if there is **negative** payoff if exercised immediately.

Example 5.3. **Moneyness of a put option**
Consider a put option with strike $K = 100$. If the inital price $S(0)$ of the underlying asset equals:

(1) 95 – then the option is in-the-money;
(2) 100 – then the option is at-the-money;
(3) 105 – then the option is out-of-the-money.

Imagine that we are half-way through the life of the option, i.e., we have reached time $T/2$. We can observe the price of the underlying asset at that time too. We denote this value by $S(T/2)$, and we can also state that at time$-T/2$

(1) if $S(T/2) > 100$ the put option is out-of-the-money;
(2) if $S(T/2) = 100$ the put option is at-the-money;
(3) if $S(T/2) < 100$ the put option is in-the-money.