

Properties of Option Prices

- American versus European
 - Since an American option can be exercised at anytime, whereas a European option can only be exercised at expiration, an American option must always be at least as valuable as an otherwise identical European option

$$C_{\text{Amer}}(S, K, T) \geq C_{\text{Eur}}(S, K, T)$$

$$P_{\text{Amer}}(S, K, T) \geq P_{\text{Eur}}(S, K, T)$$

Properties of Option Prices (price boundaries)

- Option price boundaries

- Call price cannot

- be negative
 - exceed stock price
 - be less than price implied by put-call parity using zero for put price:

$$S > C_{Amer}(S, K, T) \geq C_{Eur}(S, K, T) \geq \max[0, PV_{0,T}(F_{0,T}) - PV_{0,T}(K)]$$

- Put price cannot

- be more than the strike price
 - be less than price implied by put-call parity using zero for put price:

$$K > P_{Amer}(S, K, T) \geq P_{Eur}(S, K, T) \geq \max[0, PV_{0,T}(K) - PV_{0,T}(F_{0,T})]$$

Properties of Option Prices (early exercise)

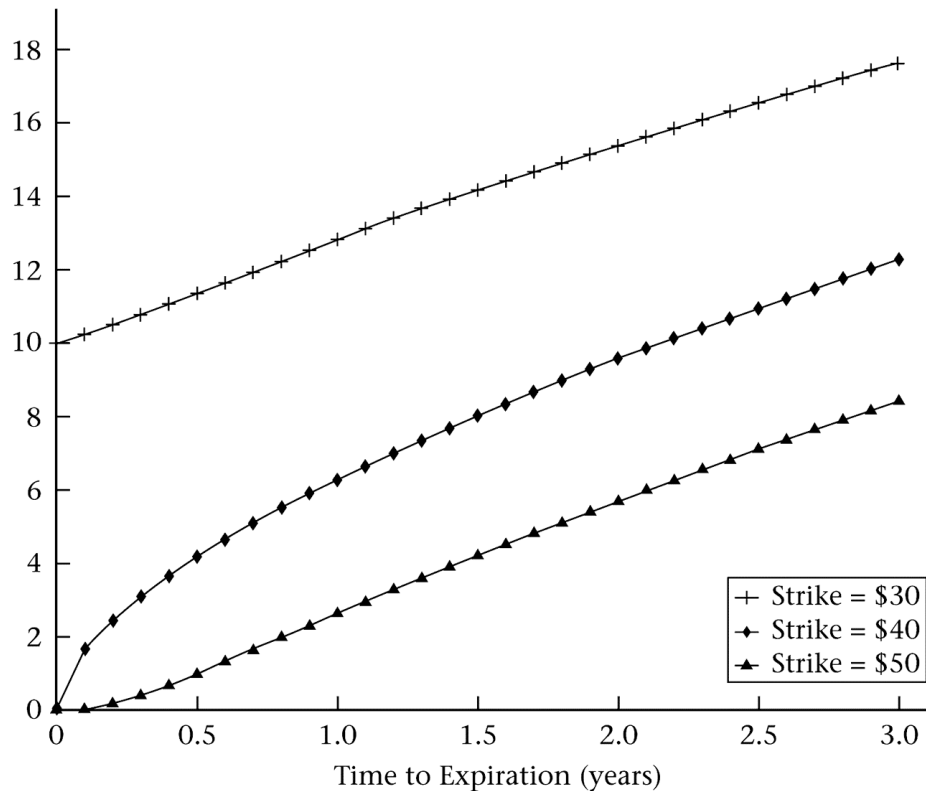
- Early exercise of American options
 - A non-dividend paying American **call** option should **not** be exercised early, because

$$C_{Amer} \geq C_{Eur} \geq S_T - K$$

- That means, one would lose money by exercising early instead of selling the option
- If there are dividends, it may be optimal to exercise early
- It may be optimal to exercise a non-dividend paying put option early if the underlying stock price is sufficiently low

Call Prices for Options As Functions of Time to Expiration I

Option Premium (\$)

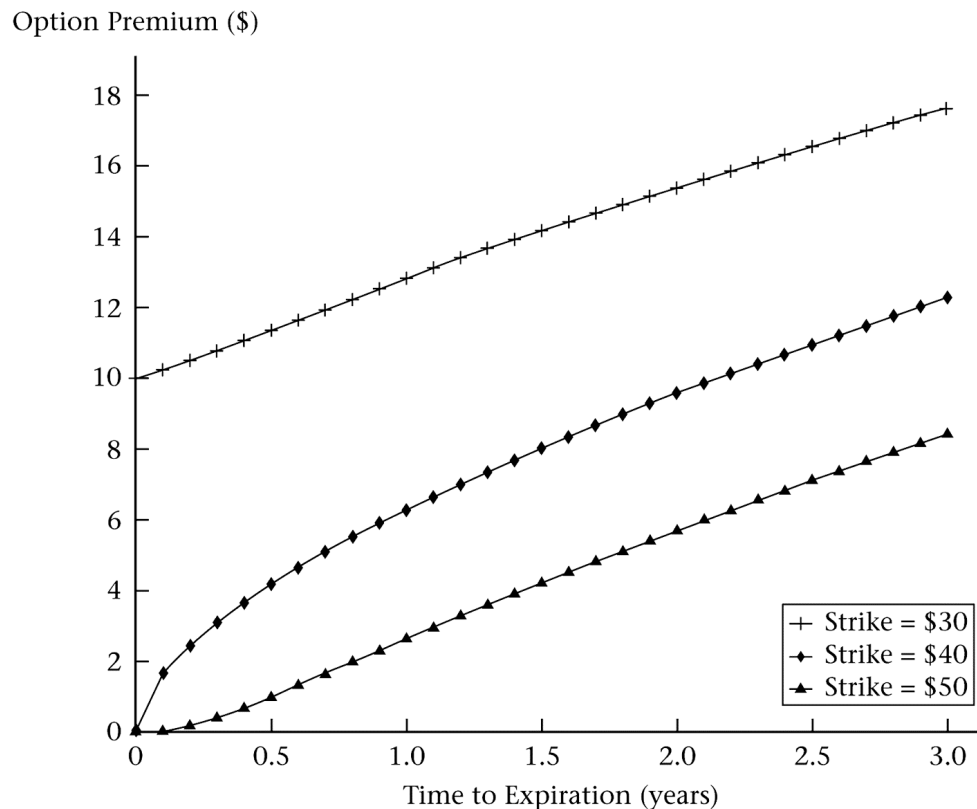


- ♦ An American option (both put and call) with more time to expiration is at least as valuable as an American option with less time to expiration. This is because the longer option can easily be converted into the shorter option by exercising it early
- ♦ A European call option on a non-dividend paying stock will be more valuable than an otherwise identical option with less time to expiration.

Call Prices for Options As Functions of Time to Expiration II

- European call options on dividend-paying stock and European puts **may be** less valuable than an otherwise identical option with less time to expiration.
- When the strike price grows at the rate of interest, European call and put prices on a non-dividend paying stock increase with time.

Different strike prices ($K_1 < K_2 < K_3$), for both European and American options (with calls on graph)



- A call with a low strike price is at least as valuable as an otherwise identical call with higher strike price

- A put with a high strike price is at least as valuable as an otherwise identical call with low strike price

-The premium difference between otherwise identical calls with different strike prices cannot be greater than the difference in strike Prices, i.e.,

$$C(K_1) - C(K_2) \leq K_2 - K_1$$

Properties of Option Prices (still, with respect to strike)

- Different strike prices ($K_1 < K_2 < K_3$), for both European and American options
 - The premium difference between otherwise identical puts with different strike prices cannot be greater than the difference in strike prices
$$P(K_1) - P(K_2) \leq K_2 - K_1$$
 - Premiums decline at a decreasing rate for calls with progressively higher strike prices. (Convexity of option price with respect to strike price - see the next graph)

$$\frac{C(K_1) - C(K_2)}{K_2 - K_1} \leq \frac{C(K_2) - C(K_3)}{K_3 - K_2}$$

Properties of Option Prices (cont'd)

