

9. Parity and Other Option Relationships

- Put-Call Parity.
- Exchange Options. Generalized Parity.

Option Comparison.

Put-Call Parity I

- The net cost of buying the index using options must equal the net cost of buying the index using a forward contract
- $\text{Call}(K, t) - \text{Put}(K, t) = \text{PV}(F_{0,t} - K)$
 - $\text{Call}(K, t)$ and $\text{Put}(K, t)$ denote the premiums of options with strike price K and time t until expiration, and $\text{PV}(F_{0,t})$ is the present value of the forward price
- *This is one of the most important relations in options!*

Put-Call Parity II

- For European options with the same strike price and time to expiration the parity relationship is

$$\text{Call} - \text{put} = PV(\text{forward price} - \text{strike price})$$

or

$$C(K, T) - P(K, T) = PV_{0,T}(F_{0,T} - K) = e^{-rT}(F_{0,T} - K)$$

- Intuition
 - Buying a call and selling a put with the strike equal to the forward price ($F_{0,T} = K$) creates a synthetic forward contract and hence must have a zero price

Put-Call Parity III

$$\text{Call } (K, T) - \text{Put } (K, T) = \text{PV } (F_{0,T} - K)$$

In words:

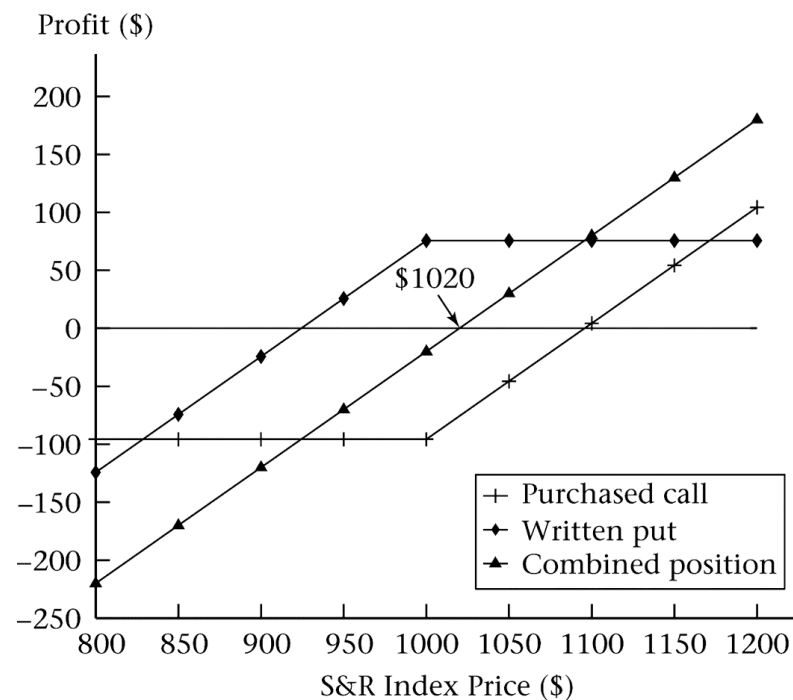
***Buying a European call + Selling a European put
(same T and K)***

is equivalent to

***making an investment in the underlying asset -
any cash payments “to” the asset until expiration***

“Synthetic” Forwards

- A synthetic long forward contract
 - Buying a call and selling a put on the same underlying asset, with each option having the same strike price and time to expiration
 - Example: buy the \$1,000-strike S&R call and sell the \$1,000-strike S&R put, each with 6 months to expiration



Parity for Options on Stocks

- If underlying asset is a stock and Div is the dividend stream, then $e^{-rT} F_{0,T} = S_0 - PV_{0,T}(Div)$, therefore

$$C(K,T) = P(K,T) + [S_0 - PV_{0,T}(Div)] - e^{-rT}(K)$$

- Rewriting above

$$S_0 = C(K,T) - P(K,T) + PV_{0,T}(Div) + e^{-rT}(K)$$

- For index options, $S_0 - PV_{0,T}(Div) = S_0 e^{-\delta T}$, therefore

$$C(K,T) = P(K,T) + S_0 e^{-\delta T} - PV_{0,T}(K)$$

Index Options

- For stock indexes it is common to model the dividend as being paid continuously at a rate **proportional** to the level of the index, i.e., the dividend yield (payment/"stock" price) is constant.
- So, for index options,

$$S_0 - PV_{0,T}(Div) = S_0 e^{-\delta T}$$

and therefore

$$C(K, T) = P(K, T) + S_0 e^{-\delta T} - PV_{0,T}(K)$$

“Synthetic” Options

- Synthetic security creation using parity:
 - Synthetic call: buy stock, buy put, borrow PV of strike and dividends
 - Synthetic put: sell stock, buy call, lend PV of strike and dividends