Basic terminology

DE:

\[ x^2 = 4 \]

\[ \rightarrow \text{ sol? } x = \pm 2 \quad (\text{can plug into eq.)} \]

Now: For future, can include operations like \( f', f'' \), ...

E.g.:

\[ y'' = 4 \]
\[ (y')^2 = 4 \]

\[ \text{sol?} \quad \rightarrow \text{must be able to plug in and satisfy equation} \]
\[ \downarrow \quad \checkmark \quad \text{Diff.} \]

<table>
<thead>
<tr>
<th>Linear</th>
<th>VS</th>
<th>Nonlinear</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ 4x = 17 ]</td>
<td>[ { \text{All var. only appear as } 1^{\text{st}} \text{ power} ]</td>
<td>[ 4x^2 = 17 ]</td>
</tr>
<tr>
<td>[ 4x_1 + x_2 = 0 ]</td>
<td>[ 4x_1 \cdot x_2 = 0 ]</td>
<td>[ 4y'_1 = 17 ]</td>
</tr>
<tr>
<td>[ 4y'_1 + y = 0 ]</td>
<td>[ 4y \cdot y' = 0 ]</td>
<td>[ 4(y')^2 = 17 ]</td>
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</table>

Always have a concept to find sol.

Case by Case
A DE is called homogeneous if every term involves a derivative of \( y \) for \( y \).

General Linear hom. DE: \[ a_n(x)y^{(n)} + a_{n-1}(x)y^{(n-1)} + \ldots + a_0(x)y = 0 \]

Interested in \( y \).

Example:

\[ 4y'' + 4y' + y = 0 \quad \text{Hom., linear w/ constant coeff.} \]

\[ = 17 \quad \text{inhom.,} \quad \ldots \]

\[ = y^{-1} \quad \text{(Hom.), nonlinear} \]

\[ 4xy'' + 4y' + 4 = 0 \quad \text{Hom., linear w/ non-const. coeff.} \]

Coeff. is a function of \( x \).

Mixed Cases: Math: \( \in \) "element of", "from the set"

\( [a, b] \):

\[
\begin{array}{c}
\text{both ends included} \\
\[ a \quad \longrightarrow \quad b \]
\end{array}
\]

\( (a, b) \):

\[
\begin{array}{c}
\text{both ends excluded} \\
\[ a \quad \longrightarrow \quad b \]
\end{array}
\]

\[ \sum_{k=1}^{n} f_k = f_1 + f_2 + \ldots + f_n \]
Some DE

1) $y' = 0$

$\Rightarrow y = \text{const.}$

General solution: $y = c$ (arbitrary $c$)

If fixed by IV if we impose some

$y = a, y = b$ is a solution (same form as $y = a$ or $y = b$)

2) $y'' = 0 \Rightarrow y' = a \Rightarrow y = ax + b$

\[
\begin{align*}
    y = ax & \rightarrow y = ax + b \\
    y = b & \\
\end{align*}
\]

the general solution

3) $y' = 4y$

$(e^{4x})' = 4e^{4x}$

$cp(x) = 4e^{-4x}$

$cp(x) = -4e^{-4x} = -4p(x)$

$(y \cdot cp)' = y' \cdot cp + y \cdot cp' = y' \cdot cp - 4y \cdot cp = (y' - 4y) \cdot cp = 0$

Thus, $y \cdot cp = \text{const.}$ i.e., $y = \frac{\text{const.}}{cp} = \text{const.} \cdot e^{4x}$