

## INFORMATION FOR SECOND EXAM

**Date:** Friday, April 1

**Time:** Usual class time (12 noon - 12:50 p.m.)

**Place:** WEL 1.308 (Same room as first exam; see map on information for first exam.)

### **What you may and may not bring to the exam:**

- You *should* bring a pencil, eraser and scratch paper.
- You *may* (and probably should) bring a *non-programmable* calculator.
- You may *not* bring notes, your textbook, or any programmable device such as a Palm Pilot, cell phone, or graphing calculator.
- You *may* (and probably should) bring your models of the Platonic Solids.

### **Exam procedures:**

- Please sit *every other seat* in the auditorium.
- Cell phones and pagers should be turned off during the exam.

**Bonus points:** You will receive 1 bonus point for every one of your Platonic Solids that you have brought to the exam, *provided:*

- You have *at least three* of them.
- They are homemade.
- You have them out and your name is attached to them when we come around to check.

**Exam coverage:** The quiz will cover Sections 2.6, 2.7, 3.1, 3.2, 3.3, 4.1, 4.5, 7.1, and 7.2 through p. 530 only.

**Exam format:** The quiz will be "semi-take-home." By that I mean that the quiz questions will be very similar to some selection from the following list of questions. By "very similar" I mean, for example, that the numbers might be changed or the wording might be different, or the setting might be different but the same techniques can be used. (The retake at the final exam time will be a slightly different selection, but also taken from the questions below.) *You are allowed to get help with these questions from anyone except me or the TA before the exam.* (You may ask me or the TA questions about anything assigned for class or covered in class, but not about the questions below or about very similar questions, unless they have been assigned in the reading, assigned as homework, or discussed in class.) As stated above, you may not bring any notes to the quiz. Also, be prepared for the questions on the quiz to be in a different order than the corresponding questions below. As stated above, you can receive bonus points for bringing at least three models of all five Platonic solids, with your name easily visible on each, and have these models set out when we come around to check them for the bonus points.

*Reminder: Unless told otherwise, you need to explain how you got your answers to receive full credit.*

## Possible Exam Questions

1. What is a rational number? Show why each of the following numbers is rational:

a.  $7\frac{3}{4}$

b. 0.0043

c. 6.4123

2. Give an example of a rational number  $x$  between  $\frac{214}{321}$  and  $\frac{215}{321}$ . Now give an example of a rational number between  $\frac{214}{321}$  and  $x$ .

3. Is the product of two rational numbers always rational, never rational, or sometimes (but not always) rational? Justify your answer by giving reasons if you say "always" or "never" and examples if you say "sometimes."

4. Is the product of two irrational numbers sometimes irrational, always irrational, or never irrational? Justify your answer by giving reasons if you say "always" or "never" and examples if you say "sometimes."

5. Explain why  $100 \times 0.1 = 10$  *without* using the rule about moving the decimal point over so many places.

6. Give an example of an irrational number between 0.11 and 0.12. Explain how you know your number is irrational.

7. Suppose you know the hypotenuse of a right triangle has length 4 inches and one leg has length 3 inches. Find the area of the triangle.

8. Can there be a right triangle with sides of length 1, 2, and 3? Why or why not? Can you find a triangle whose side lengths are consecutive whole numbers?

9. a. Express 1.262626 as a quotient of two whole numbers.

b. Express  $1.262626 \dots$  as a quotient of two whole numbers.

c. What is  $1.262626 \dots - 1.262626$ ? Express it as a decimal and also express it as a quotient of two integers.

d. Which number is larger, 1.262626 or  $1.262626 \dots$ ?

e. Find a rational number between 1.262626 and  $1.262626 \dots$ .

f. Find an irrational number between 1.262626 and  $1.262626 \dots$ .

10. True or false: If a number has an infinitely long decimal expansion, then it must be an irrational number. Explain.

11. What characterizes the decimal expansion of a rational number?

12. a. Define what is meant by "set A has the same cardinality as set B."

b. Let  $S$  be the set of all positive square roots of positive whole numbers. Exhibit (using both arrows and either words or formulas) a one-to-one correspondence between  $S$  and the set of all positive whole numbers.

c. What does your one-to-one correspondence tell you about the cardinality of  $S$ ?

13. List the positive rational numbers in one list so that the pattern of the order is clear and so that all the positive rational numbers would eventually appear on the list. Explain the pattern and why every positive rational number will eventually be on the list.

14. I have an infinite list of real numbers between zero and one. The first six numbers on my list are given below. Describe how you can construct a number between 0 and 1 whose decimal expansion consists of 3's and 0's and which is not on my list. Explain why the number you construct cannot be on my list. Give the first six digits in the decimal expansion of this number. Explain what this procedure tells us about the cardinality of the set of real numbers between 0 and 1.

First number:	0.436727384 ...
Second number:	0.728458988 ...
Third number:	0.433378349 ...
Fourth number:	0.444444444 ...
Fifth number:	0.222222222 ...
Sixth number:	0.033003303 ...

15. For each of the following numbers, determine if the number is rational or irrational. Give brief reasons justifying your answers. (In particular, if the number is rational, express it as the quotient of two whole numbers.)

a.  $3\sqrt{2}$       b. 3.14159      1.856

16. True or False: If the number  $M$  is rational, then  $1/M$  must also be rational. If you say "True," explain why the statement is true. If you say "False," give an example to show that the statement is false -- that is, give an example of a number  $M$  which is rational but whose reciprocal  $1/M$  is irrational.

17. True or False: If the number  $M$  is irrational, then  $1/M$  must also be irrational. If you say "True," explain why the statement is true. If you say "False," give an example to show that the statement is false -- that is, give an example of a number  $M$  which is irrational but whose reciprocal  $1/M$  is rational.

18. Is there a rational number between 0.999 ... and 1? Explain.

19. Is there a one-to-one correspondence between the set  $\{1, 2, 3, 4, 5\}$  and the set  $\{x, ?, \&, m, @, A\}$ ? If there is, show one. If there isn't, explain why there can't be.

20. 1. What is the probability of getting a sum greater than 4 when you roll two dice?  
(Hint: there is a hard way of doing this and an easier way.)

21. I have rolled two dice. I tell you that the sum of the two dice is 4 or less. What is the probability that the numbers on the dice sum to 3? (Drawing a chart of possibilities might help.)

22. a. What do we mean if we say that two solids are dual?

b. Fill in the following chart for the regular solids and indicate all the places on the chart where the concept of duality is shown.

	<b>Shape of faces</b>	<b># of faces</b>	<b># of edges</b>	<b># of vertices</b>	<b># of edges at each vertex</b>
<b>Tetrahedron</b>					
<b>Cube</b>					
<b>Octahedron</b>					
<b>Dodecahedron</b>					
<b>Icosahedron</b>					

c. What is the relationship between the number of faces, number of edges, and number of vertices that is true for all of these solids?

d. Why do the tetrahedron, octahedron, dodecahedron, and icosahedron have those names?