CALCULUS REVIEW PROBLEMS M 362k, Fall 03 (Smith)

1. Find $\frac{7!}{3!6!}$. (What shortcuts might you use to calculate this? Why do they work?)

2. Find $\sum_{k=2}^{6} k(k-1)$

3. Find $\sum_{i=1}^{3}\sum_{j=2}^{4} \frac{i}{j}$.

4. Write in sigma notation: 4 + 6 + 8 + 10 + 12

3. What is $\lim_{n\to\infty} \frac{1}{n}$. Why?

5. f(x) = $\begin{cases} 0, & x \le 0 \\ x, & 0 \le x \le 1 \\ 1, & 1 \le x \le 5 \\ 0, & x \ge 5 \end{cases}$

a. Find $\int_0^4 f(x)dx$ b. Find $\int_{-\infty}^{\infty} f(x)dx$

6. Find $\int_0^a x^2 e^{-x} dx$

7. Find $\int_0^\infty e^{-\frac{x}{2}} dx$

8. Find $\int_{-\infty}^{\infty} (x-1)e^{-\left(\frac{x-1}{2}\right)^2} dx$. (Clarification in case your copy is unclear: $-\left(\frac{x-1}{2}\right)^2$ is an *exponent*.)

9. Find $\frac{d}{dx}\int_0^x e^{t^2} \sin t \, dt$ [Hint: This does not take any calculation.]

10. Find $\frac{d}{dy} F(\ln(y))$ if F'(x) = e^x. (Here, ln(y) denotes the natural log of y.)

11. Find $\frac{\partial}{\partial x} e^{xy}$.

12. Use polar coordinates to calculate $2\iint_D \sqrt{R^2 - (x^2+y^2)}dxdy$ over the disk D of radius R centered at the origin. (You should get the volume of a sphere of radius R. Why?)

13. Integrate the function f(x, y) = $\begin{cases} e^{-2x-y} \text{ if } x > 0 \text{ and } y > 0 \\ 0 \quad \text{otherwise} \end{cases}$

over the region described by x < y.