Turbine Example – Revised Version

The reading of the pressure drop across an expansion valve of a turbine is expected to be influenced by gas temperature on the inlet side, operator, and the pressure gauge used by the operator. A three-way design is used to study the effects of these three factors. Three temperatures are *fixed*. Four operators and three gauges are randomly selected. Two observations are taken at each treatment level.

A three-way complete model was used.

Minitab output, with Temp fixed, other factors random:

Analysis of Variance for Drop

Source	DF	SS	MS	F	P
Temp	2	1023.36	511.68	*	
Operator	3	423.82	141.27	*	
Gauge	2	7.19	3.60	*	
Temp*Operator	6	1211.97	202.00	14.59	0.000
Temp*Gauge	4	137.89	34.47	2.49	0.099
Operator*Gauge	6	209.47	34.91	2.52	0.081
Temp*Operator*Gauge	12	166.11	13.84	0.65	0.788
Error	36	770.50	21.40		
Total	71	3950.32			

* No exact F-test can be calculated.

Compare and contrast with treating all factors random:

Source	DF	SS	MS	F	P
Temp	2	1023.36	511.68	*	
Operator	3	423.82	141.27	*	
Gauge	2	7.19	3.60	*	
Temp*Operator	6	1211.97	202.00	14.59	0.000
Temp*Gauge	4	137.89	34.47	2.49	0.099
Operator*Gauge	6	209.47	34.91	2.52	0.081
Temp*Operator*Gauge	12	166.11	13.84	0.65	0.788
Error	36	770.50	21.40		
Total	71	3950.32			

So far nothing different.

Displaying expected mean squares:

1. With Temp fixed:

Source	Variance component	Error term	Expected Mean Square (using unrestricted model)
1 Temp		*	(8) + 2(7) + 8(5) + 6(4) + Q[1]
2 Operator	-4.544	*	(8) + 2(7) + 6(6) + 6(4) + 18(2)
3 Gauge	-2.164	*	(8) + 2(7) + 6(6) + 8(5) + 24(3)
4 Temp*Operator	31.359	7	(8) + 2(7) + 6(4)
5 Temp*Gauge	2.579	7	(8) + 2(7) + 8(5)
6 Operator*Gauge	3.512	7	(8) + 2(7) + 6(6)
7 Temp*Operator*Gauge	-3.780	8	(8) + 2(7)
8 Error	21.403		(8)

* No exact F-test can be calculated.

2. With Temp random:

Source	Variance component	Error term	Expected Mean Square (using unrestricted model)
1 Temp	12.044	*	(8) + 2(7) + 8(5) + 6(4) + 24(1)
2 Operator	-4.544	*	(8) + 2(7) + 6(6) + 6(4) + 18(2)
3 Gauge	-2.164	*	(8) + 2(7) + 6(6) + 8(5) + 24(3)
4 Temp*Operator	31.359	7	(8) + 2(7) + 6(4)
5 Temp*Gauge	2.579	7	(8) + 2(7) + 8(5)
6 Operator*Gauge	3.512	7	(8) + 2(7) + 6(6)
7 Temp*Operator*Gauge	-3.780	8	(8) + 2(7)
8 Error	21.403		(8)

Note the differences in the row for the fixed factor Temp.

In this example, we see strong evidence of temperature by operator interaction, so it is not reasonable to test for the main effect of temperature. However, for purposes of illustration, if we had obtained no evidence for interaction of temperature with any of the other factors, and wanted to test the main effect of interaction, we would need to figure out and use the appropriate denominator.

From the expected mean squares table (1 above), we have

$$E(MST) = (8) + 2(7) + 8(5) + 6(4) + Q[1]$$
$$E(MSTO) = (8) + 2(7) + 6(4)$$
$$E(MSTG) = (8) + 2(7) + 8(5)$$
$$E(MSTOG) = (8) + 2(7)$$

If it were appropriate to test for main effect of T, then if the null hypothesis of no main effect of temperature were tru, we would have Q[1] = 0, and so E(MST) would equal

E(MSTO + MSTG – MSTOG). Thus MSTO + MSTG – MSTOG would be the appropriate denominator. We can get Minitab to do the test as follows:

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Approximate F-test with denominator: Temp*Operator + Temp*Gauge -
Temp*Operator*Gauge
Denominator MS = 222.63 with   7 degrees of freedom

Numerator             DF       MS      F       P
Temp                   2    511.7   2.30   0.171
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Output from a later version of Minitab:

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ANOVA: Drop versus Temp, Operator, Gauge

Factor     Type Levels Values
Temp      fixed      3    60    75    90
Operator random      4     1     2     3     4
Gauge    random      3     1     2     3


Analysis of Variance for Drop

Source                 DF         SS         MS       F      P
Temp                    2    1023.36     511.68    2.30  0.171 x
Operator                3     423.82     141.27    0.63  0.616 x
Gauge                   2       7.19       3.60    0.06  0.938 x
Temp*Operator           6    1211.97     202.00   14.59  0.000
Temp*Gauge              4     137.89      34.47    2.49  0.099
Operator*Gauge          6     209.47      34.91    2.52  0.081
Temp*Operator*Gauge    12     166.11      13.84    0.65  0.788
Error                  36     770.50      21.40
Total                  71    3950.32

x Not an exact F-test.


Source                  Variance Error Expected Mean Square for Each
Term
                       component term (using unrestricted model)
 1 Temp                           *   (8) + 2(7) + 8(5) + 6(4) + Q[1]
 2 Operator               -4.544  *   (8) + 2(7) + 6(6) + 6(4) + 18(2)
 3 Gauge                  -2.164  *   (8) + 2(7) + 6(6) + 8(5) + 24(3)
 4 Temp*Operator          31.359  7   (8) + 2(7) + 6(4)
 5 Temp*Gauge              2.579  7   (8) + 2(7) + 8(5)
 6 Operator*Gauge          3.512  7   (8) + 2(7) + 6(6)
 7 Temp*Operator*Gauge    -3.780  8   (8) + 2(7)
 8 Error                  21.403      (8)
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* Synthesized Test.

Error Terms for Synthesized Tests

Source	Error DF	Error MS	Synthesis of Error MS
1 Temp	6.97	222.63	(4) + (5) - (7)
2 Operator	7.09	223.06	(4) + (6) - (7)
3 Gauge	5.98	55.54	(5) + (6) - (7)