MORE ON SPLIT PLOT DESIGNS

The split plot model we have discussed is a special case (namely, just one block) of a more general split plot design, where the whole plots are themselves nested within blocks. If the randomization is such that each level of A appears exactly once per block and each level of B appears exactly once per whole plot, the model for this design can be expressed as

$$Y_{iujt} = \mu + \theta_h + \alpha_i + \varepsilon_{i(h)}^W + \beta_j + (\alpha\beta)_{ij} + \varepsilon_{j(hi)}^S \qquad \text{(Equation 19.2.2 in Dean-Voss)}$$

where

$h = 1, \ldots, s$; $i = 1, \ldots, a$; $j = 1, \ldots, b$

$\varepsilon_{i(h)}^W \sim N(0, \sigma_W^2)$, $\varepsilon_{j(hi)}^S \sim N(0, \sigma_S^2)$,

$\varepsilon_{i(h)}^W$'s and $\varepsilon_{j(hi)}^S$'s all mutually independent.

Here, θ_h is the effect of the h^{th} block. The sums of squares are as for the case previously discussed, except now

$$ssE_W = ssW - ss\theta - ssA.$$

The appropriate test statistics are just as before:

Null hypothesis	**Test Statistic**
H_0^A: No effect of A beyond interaction	msA/msE_W
H_0^B: No effect of B beyond interaction	msB/msE_S
H_0^{AB}: No interaction	$msAB/msE_S$

Example: (The oats example on p. 681) The experimental area was divided into s = 6 blocks, each of which was subdivided into a = 3 whole plots. Varieties of oats (factor A) were sown on whole plots according to a randomized complete block design (so every variety appeared in every block exactly once). Each whole plot was further divided into b = 4 split plots. Levels of manure were applied to the split plots according to a randomized complete block design (so each level of B appeared in each whole plot exactly once).

To run this on Minitab, enter for model:

BLOCK A WP(BLOCK) B A*B

Specify BLOCK as random. (Minitab then automatically specifies WP as random, since it is nested in BLOCK. If you try to specify WP as random, you should get an error message.)

The output is:

General Linear Model: Y versus BLOCK, A, B, WP

Factor	Type	Levels	Values
BLOCK	random	6	1 2 3 4 5 6
A	fixed	3	0 1 2
WP(BLOCK)	random	18	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18
B	fixed	4	0 1 2 3

Analysis of Variance for Y, using Adjusted SS for Tests

Source	Model DF	Reduced DF	Seq SS
BLOCK	5	5	15875.3
A	2	2	1786.4
WP(BLOCK)	12	10+	6013.3
B	3	3	20020.5
A*B	6	6	321.8
Error	43	45	7968.8
Total	71	71	51985.9

+ Rank deficiency due to empty cells or collinearity.
  No storage of results or further analysis will be done.

The last comment essentially means that we need to do the remaining calculations by hand.

The "reduced degrees of freedom" are what we need.

The test statistics are then:

For interaction:

$msAB/msE_S = (ssAB/6)/(ssE_S/45)$

$= (321.8/6)/(7968.8/45) = 53.63/177.08 = 0.30$

For an F(6, 45) distribution, this gives p-value 1 – 0.0664 = .9336.

This is consistent with no interaction.

For level of manure (factor B):

$msB/ msE_S = (ssAB/3)/(ssE_S/45)$

$= (20020.5/3)/(7968.8/45) = 6673.5/177.08 = 37.69$

For an F(3,45) distribution, this gives p-value 1 – 1.0000 = 0.0000 (to four decimal places)

This gives strong evidence that the level of manure makes a difference.

For variety of oats (factor A):

$msA/msE_W = (ssA/2)/(ssWP(BLOCK)/10)$

$= (1786.4/2)/(6013.3/10) = 893.18/601.33 = 1.49.$

For an $F(2,10)$ distribution, this gives p-value $1 – 0.7286 = .2714$.

This is consistent with no effect of variety.

A further analysis would involve contrasts; see pp. 683 – 684.

Note that $msE_S = 177.08$ (the estimate of error variance for whole plots) is noticeably smaller than $msE_W = 601.33$ (the estimate of error variance for split plots), as is typical of split plot designs.