RANDOM EFFECTS MODELS (Chapter 17)

So far we have only studied experiments and models with only *fixed effect* factors: factors whose levels have been specifically fixed by the experimenter, and where the interest is in comparing the response for just these fixed levels.
A *random effect* factor is one which has many possible levels, where the interest is in the variability of the response over the entire population of levels, but we only include a random sample of levels in the experiment.

Examples: Classify as fixed or random effect.

1. The purpose of the experiment is to compare the effects of three specific dosages of a drug on response.

2. A textile mill has a large number of looms. Each loom is supposed to provide the same output of cloth per minute. To check whether this is the case, five looms are chosen at random and their output is noted at different times.

3. A manufacturer suspects that the batches of raw material furnished by his supplier differ significantly in zinc content. Five batches are randomly selected from the warehouse and the zinc content of each is measured.

4. Four different methods for mixing Portland cement are economical for a company to use. The company wishes to determine if there are any differences in tensile strength of the cement produced by the different mixing methods.

Note: The theory behind the techniques we discuss assumes that the population of levels of the random effect factor is infinite. However, the techniques fit well as long as the population is at least 100 times the size of the sample being observed. Situations where the population/sample size ratio is smaller than 100 require "finite population" methods which we will not cover in this class.

The Random-Effects One-Way Model

For a completely randomized design, with v random selected levels of a treatment factor T, and t observations for each of these v levels, we can use the model

$$Y_{it} = \mu + T_i + \varepsilon_{it},$$

where:

Each $\varepsilon_{it} \sim N(0, \sigma^2)$
The ε_{it}'s are independent random variables
The T_i's are independent random variables with distribution $N(0, \sigma_T^2)$
The T_i's and ε_{it}'s are independent of each other.

Caution: The terminology can be confusing. Here is how to think of the model: