RCBD EXAMPLE

Example: A hardness testing machine operates by pressing a tip into a metal test "coupon." The hardness of the coupon can be determined from the depth of the resulting depression. Four tip types are being tested to see if they produce significantly different readings. However, the coupons might differ slightly in their hardness (for example, if they are taken from ingots produced in different heats). Thus coupon is a nuisance factor, which can be treated as a blocking factor. Since coupons are large enough to test four tips on, a RCBD can be used, with one coupon as a block. Four blocks were used. Within each block (coupon) the order in which the four tips were tested was randomly determined. The results (readings on a certain hardness scale) are shown in the following table:

	Test Coupon			
Type of Tip	**1**	**2**	**3**	**4**
1	9.3	9.4	9.6	10.0
2	9.4	9.3	9.8	9.9
3	9.2	9.4	9.5	9.7
4	9.7	9.6	10.0	10.2

Comment: From the table, the type of design is not apparent – in particular, the table does not show the order in which the observations were made, hence does not show the randomization. However, data are often presented in such a table, for reasons of economy of space or whatever.

We wish to test

H_0: All tips give the same mean reading

against the alternative

H_a : At least two tips give different mean readings.

Our pre-planned analysis will be to test this hypothesis at the .01 level, then if the hypothesis is rejected, to form confidence intervals for pairwise differences at a family rate of 99%, giving an overall confidence/significance level of 98%.

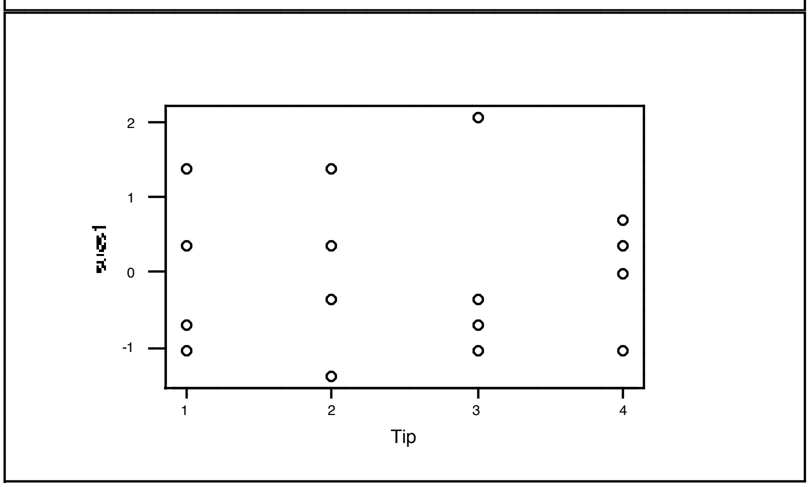
We can run the data on Minitab under Balanced ANOVA in exactly the same way we would run a two-way main effects model. The output is:

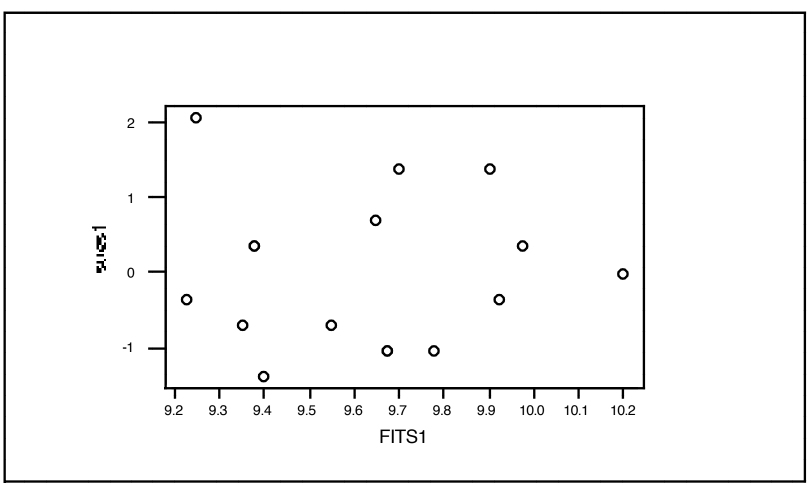
```
Analysis of Variance for hard

Source      DF        SS         MS        F      P
Coupon       3   0.82500    0.27500    30.94  0.000
Tip          3   0.38500    0.12833    14.44  0.001
Error        9   0.08000    0.00889
Total       15   1.29000
```

Note that degrees of freedom and sums of squares behave as expected.

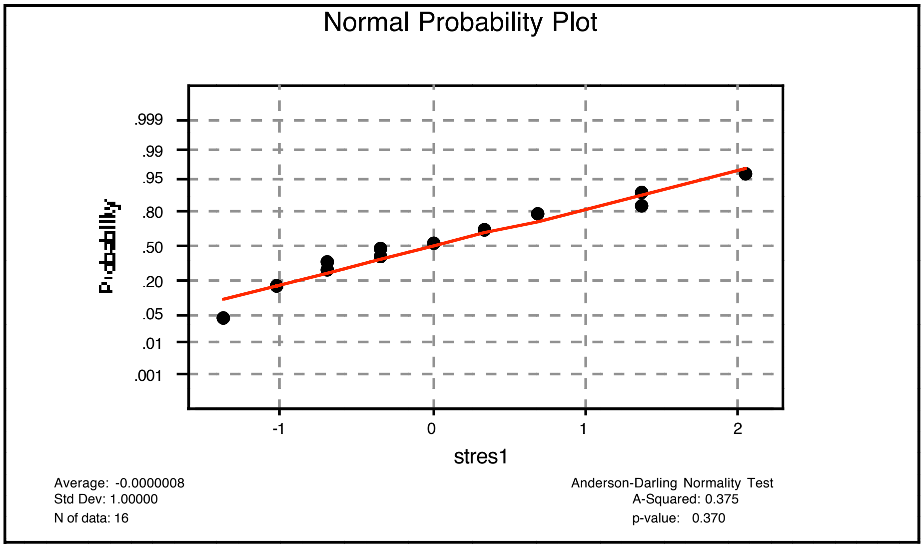
Before testing, we check the model.

The plots of standardized residuals vs blocks, factor levels, and fits:

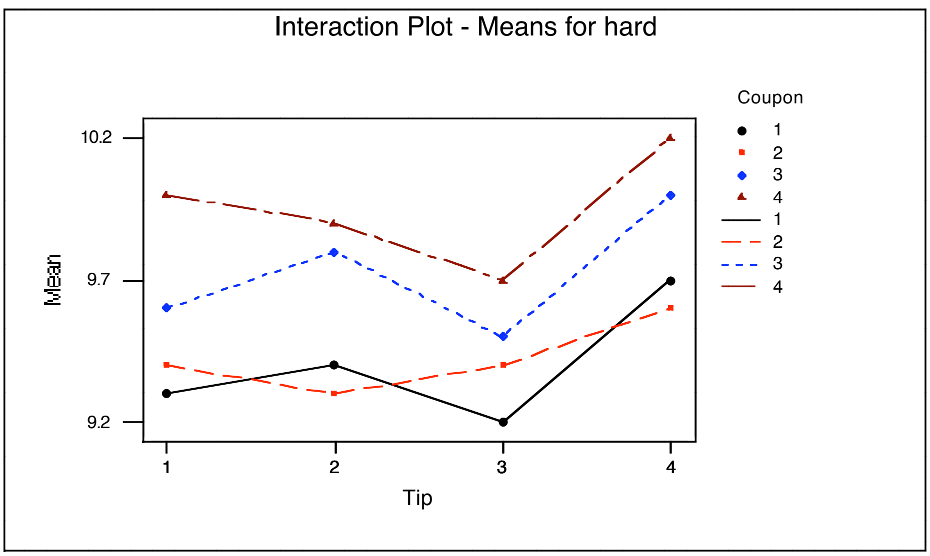




Normal probability plot:



Plot of y_{hi}'s vs i, marked by block:



Are there any concerns from the plots that should cause us not to proceed with inference or to proceed with caution?

If we decide to proceed with inference:

The p-value for our hypothesis test is 0.001, prompting us to reject the null hypothesis of no difference at our pre-planned .01 significance level. (Are there any cautions or reservations coming from the model checking?)

The F-statistic and p-value shown in the "coupon" row have no interpretation for inference. However, the large ratio of msCoupon to msE suggests that blocking has resulted in significant reduction in variance.

Exercise 1: Suppose we used four coupons, randomly assigned the tips to each (so obtained a completely randomized design with single factor Tip), and by chance obtained the same results as in the block design experiment. Analyze the data under this assumption and compare with the results in the RCBD analysis.

We proceed to form confidence intervals for differences in effect of tip. (Note that a glance at the data suggests that tip 4 tends to give higher readings; we will see whether or not the confidence intervals suggest that this is more than just chance variability.) Note (see more below) that we *cannot* use Minitab's option of obtaining the CI's doing one-way analysis of variance – the msE is wrong. The Tukey msd is

$$\text{msd} = w_T\sqrt{msE\left(\frac{1}{4}+\frac{1}{4}\right)} = [q(4, 9, 0.01)/\sqrt{2}\,]\sqrt{0.00889\left(\frac{1}{2}\right)}$$
$$= (5.96/\sqrt{2})(0.0667) = 0.281$$

Using Descriptive Statistics, we calculate the estimates $\bar{y}_{\bullet i}$ to be 9.575, 9.600, 9.450, and 9.875, for i = 1, 2, 3, and 4, respectively. So the centers of the simultaneous 98% confidence intervals for the pairwise difference contrasts are:

Contrast	**Center of CI**
τ_1-τ_2	9.575 – 9.600 = -0.025
τ_1-τ_3	9.575 – 9.450 = 0.125
τ_1-τ_4	9.575 – 9.875 = -0. 300
τ_2-τ_3	9.600 – 9.450 = 0.150
τ_2-τ_4	9.600 – 9.875 = -0.275
τ_3-τ_4	9.450 – 9.875 = - 0.425

Comparing with the msd, we can see that we have the mean for tip 4 significantly different from the means for the other tips, but no significant differences between the means of the other three tips. This is what we suspected from the data.

Exercises:
2. Investigate Bonferroni and Scheffe methods to see if they might give smaller confidence intervals.

3. Find the confidence intervals under the assumptions of Exercise 1.

Be sure to read Examples 10.4.1 and 10.4.2, and Section 10.5 for more examples of RCBD's and their analysis