ANALYSIS OF SPLIT PLOT DESIGNS

Note: Our model

$$Y_{iujt} = \mu + \alpha_i + \varepsilon_{iu}^{W} + \beta_j + (\alpha\beta)_{ij} + \varepsilon_{jt(iu)}^{S}$$

is the simplest form of split-plot design. The more general form discussed in the book also has blocks containing the whole plots. There are also random effects and mixed effects forms of split-plot designs, and forms incorporating more than two factors.

As suggested by the form of the model, the analysis combines two separate analyses: the whole plot analysis and the split-plot analysis. The idea is that the whole plots act like blocks for the split plot analysis. The sum of squares for whole plots, ssW, is calculated in a similar fashion to the sum of squares for blocks in a randomized complete block design. The whole plot error sum of squares is then

$$ssE_W = ssW - ssA.$$

The split plot error sum of squares is

$$ssE_S = sstot - ssW - ssB - ssAB.$$

Each has an associated degree of freedom. Mean squares are defined as sums of squares divided by degrees of freedom. The test statistics are:

Null hypothesis	**Test Statistic**
H_0^A: No effect of A beyond interaction	msA/msE_W
H_0^B: No effect of B beyond interaction	msB/msE_S
H_0^{AB}: No interaction	$msAB/msE_S$

To run on Minitab and many other programs, use the following trick: Create a new variable (usually called W or WP) which indicates which whole plot each observation belongs to. (Use 1, 2, … , a*l* to label the whole plots.) In General Linear Model, declare this variable random. In specifying factors, indicate that this factor is nested in A (the whole plot factor).

Example: In the experiment studying the effect of pretreatment and stain on water resistance, the data (including W) are as shown:

pretreat	stain	resist	W
2	2	53.5	4
2	4	32.5	4
2	1	46.6	4
2	3	35.4	4
2	4	44.6	5
2	1	52.2	5
2	3	45.9	5
2	2	48.3	5

1	3	40.8	1
1	1	43.0	1
1	2	51.8	1
1	4	45.5	1
1	2	60.9	2
1	4	55.3	2
1	3	51.1	2
1	1	57.4	2
2	1	32.1	6

2	4	30.1	6
2	2	34.4	6
2	3	32.2	6
1	1	52.8	3
1	3	51.7	3
1	4	55.3	3
1	2	59.2	3

In Minitab, use General Linear Model.

Response: resist

Model: pretreat W(pretreat) stain pretreat* stain

Random: W

The output is:

General Linear Model: resist versus pretreat, stain, W

```
Factor        Type Levels Values
pretreat     fixed      2 1 2
W(pretreat) random      6 1 2 3 4 5 6
stain        fixed      4 1 2 3 4

Analysis of Variance for resist, using Adjusted SS for Tests

Source           DF     Seq SS     Adj SS     Adj MS       F      P
pretreat          1     782.04     782.04     782.04    4.03  0.115
W(pretreat)       4     775.36     775.36     193.84   15.25  0.000
stain             3     266.00     266.00      88.67    6.98  0.006
pretreat*stain    3      62.79      62.79      20.93    1.65  0.231
Error            12     152.52     152.52      12.71
Total            23    2038.72
```

Note:
1. We ignore the P-value for W.
2. This does not work with Minitab 10.
3. ssE_W is in the line W(pretreat).
4. ssE_S is in the line Error
5. Check that the sums of squares add as indicated above.
6. Check that the test ratios are as they should be.
7. Note that ssE_W is much larger than ssE_S. This is typical. Why?
8. If we don't designate W as random, we get different output:

General Linear Model: resist versus pretreat, stain, W

```
Factor        Type Levels Values
pretreat     fixed      2 1 2
W(pretreat)  fixed      6 1 2 3 4 5 6
stain        fixed      4 1 2 3 4

Analysis of Variance for resist, using Adjusted SS for Tests

Source           DF     Seq SS     Adj SS     Adj MS       F      P
pretreat          1     782.04     782.04     782.04   61.53  0.000
W(pretreat)       4     775.36     775.36     193.84   15.25  0.000
stain             3     266.00     266.00      88.67    6.98  0.006
pretreat*stain    3      62.79      62.79      20.93    1.65  0.231
Error            12     152.52     152.52      12.71
Total            23    2038.72
```

What is different? How do we know the first method is the one we want?