SPLIT PLOT DESIGNS

Key Features: More than one type of experimental unit and more than one randomization.

Typical Use: When one factor is difficult to change.

Example (and terminology): An agricultural researcher us studying the effects of corn variety and irrigation level on corn yields. Four varieties of corn and three irrigation levels are used. It is easy to plant a specified variety of corn wherever told to do so. (Variety is an easy-to-change factor.) However, irrigation is done by large sprinklers that irrigate a large area of land. (Irrigation is a difficult-to-change factor.) So a crossed factorial experiment with just two replications of each of the twelve treatment combinations would require 24 large areas of land. This is beyond resources available.

To get around the problem: Use just six plots of land, chosen so that each can be irrigated at a set level without affecting the irrigation of the others. (These large plots are called the *whole plots.*) Randomly assign irrigation levels to each whole plot to have two plots for each irrigation level. (Irrigation is called the *whole-plot factor*). Divide each whole plot into four subplots. (Each subplot is called a *split plot.*) Within each whole plot, randomly assign the four corn varieties to the four split plots. (Variety is called the *split-plot factor*.)

1. Draw a picture to illustrate the design.

2.What are the two experimental units and the corresponding two randomizations?

3. How is this design like a two-way crossed factorial design? How are the two designs different?

4. How is this like a randomized complete block design? How are the two designs different?

5. Does the split-plot design introduce any possible confounding?

Second example: An industrial experimenter is studying how the water resistance of wood depends on the pretreatment (two types) and the stain (four types). It turns out to be very difficult to apply the pretreatment to a small wood panel, so instead each type of pretreatment is applied to a whole board, the board is then cut into four smaller wood panels, and one type of stain is applied to each panel. Six whole boards are used. Give details to make this into a split-plot design. Identify whole and split plots and whole-plot and split-plot factors. Is there any confounding?

Note: The book discusses the more complex situation where whole plots are within blocks.

Model for Split-Plot Designs

A split-plot experiment can be considered as two experiments superimposed: One experiment has the whole-plot factor applied to the large experimental units (whole plots), and the other experiment has the split-plot factor applied to the smaller experimental units (split plots). The model will reflect this by including two error terms: the whole-plot random error ε^W and the split-plot random error ε^S. To get subscripts straight, we will need to set notation:

- A (with a levels) is the whole-plot factor.
- B (with b levels) is the split-plot factor.
- Each level of A is assigned to *l* whole plots. (So there are a*l* whole plots.)
- Each level of B is assigned to m split plots.

Exercise: What are a, b, *l*, and m in the examples?

The model (Equation 19.2.3 on p. 678) is given on two lines to emphasize the two error terms:

$$
\begin{aligned}
Y_{iujt} = \mu + \alpha_i + \ & \varepsilon_{iu}{}^W \\
& + \beta_j + (\alpha\beta)_{ij} + \varepsilon_{jt(iu)}{}^S
\end{aligned}
$$

where

$i = 1, \ldots, a;\ u = 1, \ldots, l;\ j = 1, \ldots, b;\ t = 1, \ldots, m;$

$\varepsilon_{iu}{}^W \sim N(0, \sigma_W{}^2)$, $\varepsilon_{jt(iu)}{}^S \sim N(0, \sigma_W{}^2)$,

$\varepsilon_{iu}{}^W$'s and $\varepsilon_{jt(iu)}{}^S$'s all mutually independent.

Exercise: Why is the nesting notation in the subscript of the split-plot error used?