APPROXIMATE F-TESTS FOR RANDOM FACTOR MODELS

Recall: In the three-way random complete effects model, the usual method did not give us a way to test H_0^A: $\sigma_A^2 = 0$.

Expected mean squares:

$E(MSA) = rbc\sigma_A^2 + rc\sigma_{AB}^2 + rb\sigma_{AC}^2 + r\sigma_{ABC}^2 + \sigma^2$

E(MSB) and E(MSC) similar

$E(MSAB) = rc\sigma_{AB}^2 + r\sigma_{ABC}^2 + \sigma^2$

E(MSBC) and E(MSAC) similar

$E(MSABC) = r\sigma_{ABC}^2 + \sigma^2$

$E(MSE) = \sigma^2$.

If H_0^A: $\sigma_A^2 = 0$ is true, then

$E(MSA) = rc\sigma_{AB}^2 + rb\sigma_{AC}^2 + r\sigma_{ABC}^2 + \sigma^2$,

which is *not* the expected value of any of the mean squares above.

But we can use the ideas we used in finding unbiased estimators of mean squares to find some sum of mean squares with this same expected value. In fact,

E(MSAB + MSAC – MSABC) =

=

as desired.

This suggests (MSA)/(MSAB + MSAC – MSABC) as a test statistic.

To see the general pattern, let

U = MSAB + MSAC – MSABC

$= \Sigma k_i (MS)_i$.

Note that the equation above says that if H_0^A: $\sigma_A^2 = 0$ is true, then E(U) = E(MSA).

As in our discussion of approximate confidence intervals, the model assumptions imply that

$$xU/E(U) \approx \chi^2(x),$$

where

$$x = \frac{\left(\sum k_i (ms)_i\right)^2}{\sum k_i^2 (ms)_i^2 / x_i}.$$

Also, if H_0^A is true, the model assumptions imply that

- MSA/E(MSA) (i.e., MSA/E(U)) is χ^2(a-1)/(a-1)
- MSA/E(MSA) and U/E(U) are independent

Thus

$$MSA/U = [MSA/E(U)]/[U/E(U)] \approx F(a\text{-}1, x),$$

where a non-integral value of df for F makes sense in terms of a formula for the pdf with degrees of freedom as parameters.

This gives us an *approximate* F-test for H_0^A.

Comment: There are other possible test statistics – e.g., [MSA + MSABC]/[MSAB + MSAC] could be reasoned to be a test statistic for H_0^A. But to get an F-distribution, we need numerator and denominator independent.

<u>Example</u>: The reading of the pressure drop across an expansion valve of a turbine is expected to be influenced by gas temperature on the inlet side, operator, and the pressure gauge used by the operator. A three-way random effects experiment is used to study the effects of these three factors. Three temperatures, four operators, and three gauges are randomly selected. Two observations are taken at each treatment level. A three-way complete model is used.

Analysis of Variance for Drop

Source	DF	SS	MS	F	P
Temp	2	1023.36	511.68	*	
Operator	3	423.82	141.27	*	
Gauge	2	7.19	3.60	*	
Temp*Operator	6	1211.97	202.00	14.59	0.000
Temp*Gauge	4	137.89	34.47	2.49	0.099
Operator*Gauge	6	209.47	34.91	2.52	0.081
Temp*Operator*Gauge	12	166.11	13.84	0.65	0.788
Error	36	770.50	21.40		
Total	71	3950.32			

- No exact F-test can be calculated.

Approximate F-test for “main effect” of gauge:

Approximate F-test with denominator:
Temp*Gauge + Operator*Gauge - Temp*Operator*Gauge

Denominator MS = 55.542 with 6 degrees of freedom

Numerator	DF	MS	F	P
Gauge	2	3.597	0.06	0.938

Comments: There is some controversy as to which of alternate tests are best; e.g., some authors indicate that the approximation works better when all coefficients in denominators are positive. Different software may have different tests.