

GENERAL COMPLETE BLOCK DESIGNS (GCBD's)

Recall: In a Randomized Complete Block Design, each treatment occurs only once in each block. Consequently, it is not possible to test for interaction between block and treatment. General Complete Block Designs allow replications of treatments within blocks.

Notation and Experimental Design for a GCBD:

v treatments (They might be factorial treatment combinations)

b blocks

$k = vs$ experimental units in each block

The units in each block are randomly assigned to the v treatments in such a way that each treatment is assigned s units per block

Total number of experimental units $n =$

Compare and contrast with RCBD.

Models for GCBD's:

1. Block-treatment model:

$$Y_{hit} = \mu + \theta_h + \tau_i + \varepsilon_{hit}$$

$$\varepsilon_{hit} \sim N(0, \sigma^2)$$

ε_{hit} 's independent

where

Y_{hit} is the random variable representing the response for observation t of treatment i observed in block h ,

μ is a constant (which may be thought of as the overall mean – see below)

θ_h is the (additive) effect of the h^{th} block ($h = 1, 2, \dots, b$)

τ_i is the (additive) effect of the i^{th} treatment ($i = 1, 2, \dots, v$)

ε_{hit} is the random error for the observation t of the i^{th} treatment in the h^{th} block.

Compare and contrast with RCBD model, main effects model.

Does not model interaction between block and factor.

2. Block-treatment interaction model

$$Y_{hit} = \mu + \theta_h + \tau_i + (\theta\tau)_{hi} + \varepsilon_{hit}$$

$$\varepsilon_{hit} \sim N(0, \sigma^2)$$

ε_{hit} 's independent

where

Y_{hit} , μ , θ_h , τ_i , ε_{hit} are as above, and $(\theta\tau)_{hi}$ is a *block-treatment* interaction term.

Compare and contrast with block-treatment model, 2-way complete model.

Does model interaction between block and factor.

Fitting the model: Least squares, etc. gives fits:

$$\text{Block-treatment model: } \hat{y}_{hit} = \bar{y}_{h..} + \bar{y}_{.i} - \bar{y}_{...}$$

$$\text{Block-treatment interaction model: } \hat{y}_{hit} = \bar{y}_{hi}$$

Residuals, ssE, etc. are defined as usual.

Model checking: As for RCBD. Bear in mind that non-parallel lines in an interaction plot could suggest either interaction or large error variability.

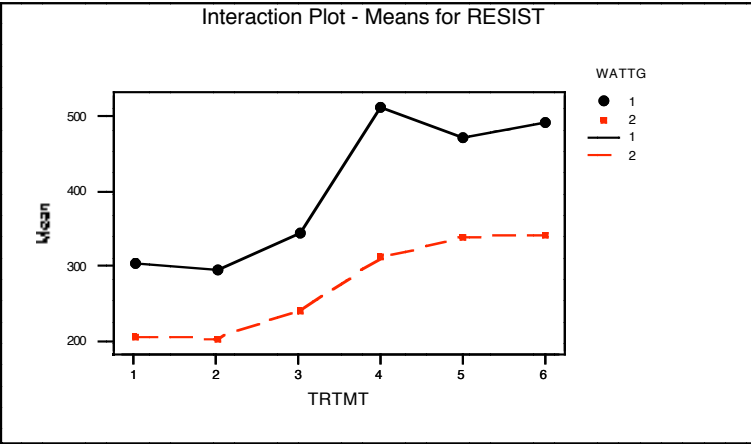
Analysis: The analyses for the two models look almost like the analyses for the two-way main effects model and the two-way complete model, respectively, and can be run on software using the same routines. *But be sure to remember the difference:* there is no test for block, although the msB to msE ratio is an informal assessment of the utility of blocking in reducing error variance.

In particular, in the block-treatment interaction model, the null hypothesis for the test for interaction between block and treatment is

$$H_0^{\theta T} : [(\theta\tau)_{hi} - [(\theta\tau)_{hs}] - [(\theta\tau)_{gi} - [(\theta\tau)_{gs}]] \text{ for all } h \neq g, i \neq s$$

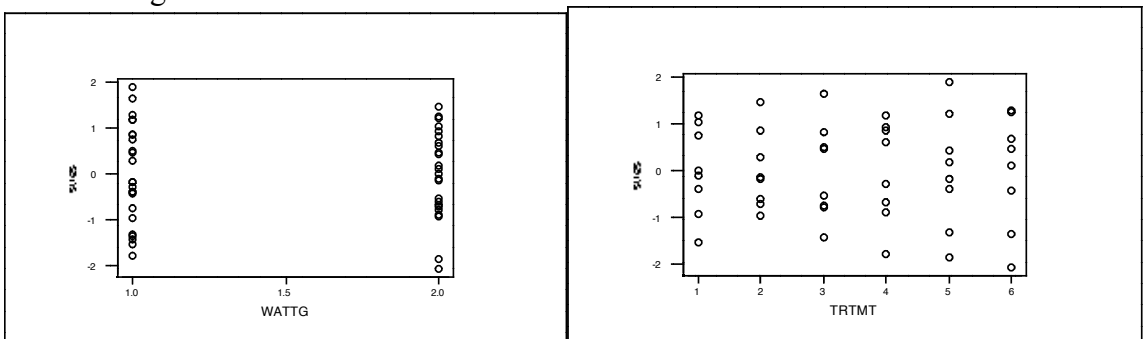
Contrasts: Methods for multiple comparison used for factorial designs are valid with suitable modifications; see pp. 312 – 313 for details.

Example: Light bulb experiment: The purpose of the experiment was to compare the light intensities of three different brands (coded 1, 2, 3) of light bulbs to determine the best brand. (Brand 3 was cheaper than brands 1 and 2.) A second treatment factor was percent of capacity, which was set at two levels, 50% and 100%, by adjusting the current passed through the bulbs. The experimenters wished to compare brands averaged across capacities, and also for each capacity. Both 60 watt and 100 watt bulbs were to be compared across brands, but comparisons between wattages were not of interest. Also, since the experiment needed to be run at two different times, it was convenient to run the 60 watt bulbs one day and the 100 watt bulbs the other day. Thus the watt-day combinations were used as blocks. In each block, 4 observations were taken on each of the 6 treatments. Resistance was the response, since low resistance implies high illumination. It was reasonable to believe that there might be interaction between treatment combination and wattage (hence block), so the block-treatment interaction model was used. Indeed, an interaction plot for treatment and block suggested possible interaction.

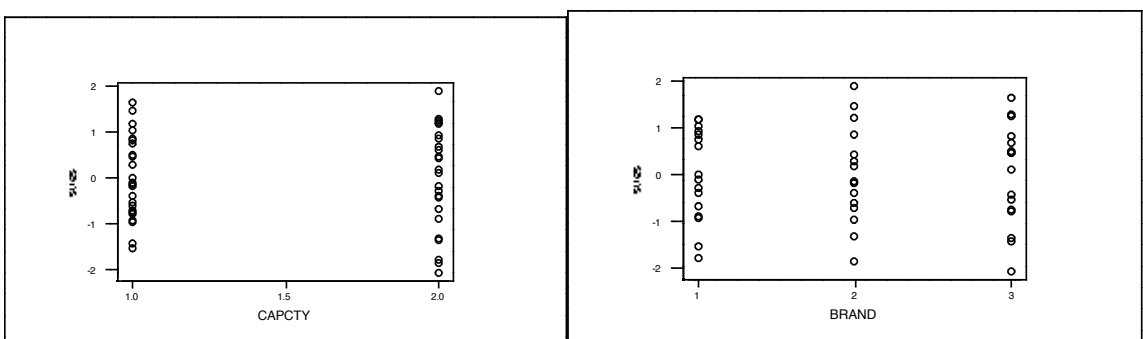


Model-checking plots of standardized residuals from the block-treatment interaction model are shown below. Are there any indications of problems with the model?

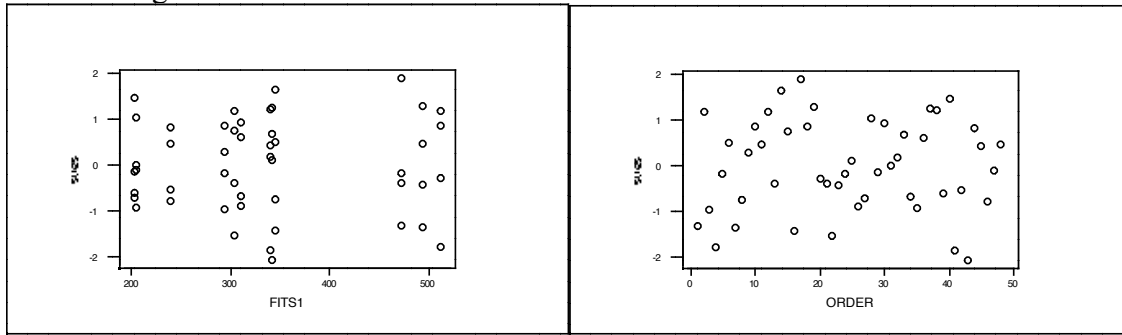
Residuals against block and treatment:



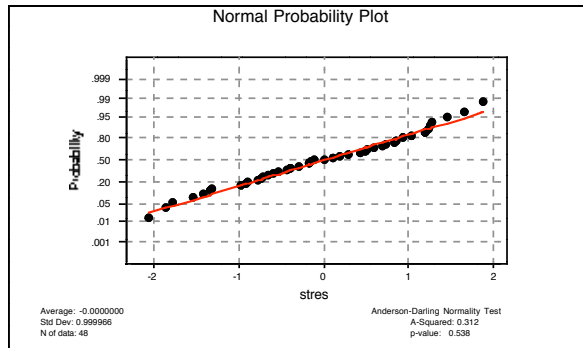
Residuals against the two treatment factors:



Residuals against fits and order:



A normal plot:



The Analysis of Variance table:

Analysis of Variance for RESIST

| Source | DF | SS | MS | F | P |
|-------------|----|--------|--------|---------|-------|
| WATTG | 1 | 203972 | 203972 | 2025.79 | 0.000 |
| TRTMT | 5 | 267645 | 53529 | 531.64 | 0.000 |
| WATTG*TRTMT | 5 | 17503 | 3501 | 34.77 | 0.000 |
| Error | 36 | 3625 | 101 | | |
| Total | 47 | 492745 | | | |

The data do provide evidence for interaction between block and treatment. The high ratio of msB/msE also suggests that blocking has been useful in reducing variance.

The original questions of interest (comparing brands averaged across capacity and comparing brands for each capacity) call for contrasts. However, the interaction between block and treatment suggests that these comparisons should be made for each block (each wattage) separately. See Example 10.6.2 (pp. 313 – 315) for details of the further analysis.

OTHER BLOCK DESIGNS

Blocking in factorial experiments: When treatments in a block design are combinations of two or more factors, we can use models that involve the individual factors and their interactions as well as the blocking “factor” and its interactions with the factors. See Section 10.8 for examples.

Incomplete block designs: In these, the number of treatments per block is not a multiple of the total number of treatments – most commonly, it is less than the total number of treatments. This may be necessary because of limits on the size of the blocks, or because of limitations on availability of equipment, etc. In *balanced incomplete block designs*, each block has the same number of treatments, but the number of treatments per block is less than the total number of treatments. For more on incomplete block designs, see Chapter 11.