TESTING FOR TREATMENT EFFECT VARIANCE AS A PROPORTION OF ERROR VARIANCE

Sometimes when using a random effects model, it is of interest to test whether the random effects variance component is less than or equal to some proportion of the error variance. So the null and alternate hypotheses are:

$$H_0^{\gamma T}: \sigma_T^2 \le \gamma\sigma^2 \quad \text{and} \quad H_a^{\gamma T}: \sigma_T^2 > \gamma\sigma^2$$

However, this is not a suitable set-up for the logic of hypothesis testing. So instead we really do the test with null and alternate hypotheses

$$H_{00}^{\gamma T}: \sigma_T^2 = \gamma\sigma^2 \quad \text{and} \quad H_a^{\gamma T}: \sigma_T^2 > \gamma\sigma^2,$$

and reject (respectively, fail to reject) $H_0^{\gamma T}$ if we reject (respectively, fail to reject) $H_{00}^{\gamma T}$.

The reasoning behind the second test: (Compare and contrast with the reasoning for the test for $H_0^T: \sigma_T^2 = 0$.)

If $H_{00}^{\gamma T}$ is true, then E(MST) = $c\sigma_T^2 + \sigma^2 = c\gamma\sigma^2 + \sigma^2 = (c\gamma + 1)\sigma^2$. Thus if $H_{00}^{\gamma T}$ is true, the ratio MST/MSE should be about $c\gamma + 1$ (since E(MSE) = σ^2).

Similarly, if $H_a^{\gamma T}$ is true, then E(MST) > $(c\gamma + 1)\sigma^2$, so MST/MSE would tend to be > $c\gamma + 1$.

It can be shown mathematically that

$$\frac{SST \big/ \left((c\sigma_T^2 + \sigma^2)(v-1)\right)}{SSE \big/ \left(\sigma^2(n-v)\right)} \sim F(v-1, n-v).$$

This fraction reduces to $\dfrac{MST \big/ \left(c\sigma_T^2 + \sigma^2\right)}{MSE \big/ \left(\sigma^2\right)}$. If $H_{00}^{\gamma T}$ is true, then the fraction further reduces to $\dfrac{MST \big/ (c\gamma + 1)}{MSE} = \dfrac{MST}{MSE(c\gamma + 1)}$. This then says that $\dfrac{MST}{MSE(c\gamma + 1)} \sim F(v-1, n-v)$. So if $\dfrac{MST}{MSE(c\gamma + 1)}$ is large compared with F(v-1, n-v), this would give evidence against $H_{00}^{\gamma T}$.

Example: In the loom experiment, suppose we want to test whether variability due to loom is greater than error variability (i.e., $\gamma = 1$).