ESTIMATING VARIANCE COMPONENTS FOR RANDOM EFFECTS (Sections 17.6.4 and 17.6.5)

Unbiased estimators: Two examples will illustrate the method.

I. In the two-way complete model:

1. Recall that $E(MSE) = \sigma^2$, so MSE is an unbiased estimator for σ^2.

2. From $E(MSAB) = r\sigma_{AB}^2 + \sigma^2$ and $E(MSE) = \sigma^2$, we get

$$E\left(\frac{MSAB - MSE}{r}\right) = \sigma_{AB}^2$$

3. From the above and $E(MSA) = br\sigma_A^2 + r\sigma_{AB}^2 + \sigma^2$, we get

$$E\left(\frac{MSA - MSAB}{br}\right) = \sigma_A^2.$$

4. Similarly, $E\left(\frac{MSB - MSAB}{ar}\right) = \sigma_B^2$

II. Find an unbiased estimator of σ_B^2 in the model with model equation

$$Y_{ijkt} = \mu + A_i + B_j + C_k + (AB)_{ij} + (BC)_{jk} + \varepsilon_{ijt}$$

Confidence intervals:

First, use the method above to obtain an unbiased estimator U of σ_*^2 (where $*$ = A, B, AB, etc.).

U will be some linear combination of mean squares – say $U = \Sigma k_i(MS)_i$.

Let $(ms)_i$ be the value of $(MS)_i$ from the sample obtained in the experiment.

Let x_i be the degrees of freedom of $(MS)_i$.

Then it can be proved mathematically (using model assumptions) that $xU/\sigma_*^2 \approx \chi^2(x)$, where

$$x = \frac{\left(\sum k_i (ms)_i\right)^2}{\sum k_i^2 (ms)_i^2 / x_i}.$$

We can use this, following the method we used to find an approximate CI for σ_T^2 in the one-random effects model, to get an approximate CI for σ_*^2.

Example: With the fiber breaking strength data, find an approximate 95% CI for σ_A^2, where A is operator.

We have $E([MSA-MSAB]/(4\cdot 2)) = \sigma_A^2$, so

$U = (1/8)MSA - (1/8)MSAB$

Then …

Questions:

Comment on the size of this CI.

Is it symmetric about the point estimate?

Variance components on Minitab: Run the fiber data, but clicking "Storage" (or "Options") and "Display expected mean squares."

```
Source              Variance Error Expected Mean Square
                   component  term (using unrestricted
model)
 1 Operator           9.0903   3   (4) + 2(3) + 8(1)
 2 Machine           -0.5486   3   (4) + 2(3) + 6(2)
 3 Operator*Machine   1.8264   4   (4) + 2(3)
 4 Error              3.7917       (4)
```

a. Note "Variance components" – does the value for Operator agree with the above calculation?

b. Note the variance component for Machine. Is this good or bad?

What does this have to do with random variability?

How does this relate to the p-value in the output?

c. Where does most of the variance come from?

d. Recall the hypothesis tests:

For Operator and Machine – denominator was MSAB – item 3.

For Operator*Machine – denominator was MSE – item 4.

e. The last column contains a shorthand for the expected mean squares.

For example, recall that in this example, $E(MSA) = br\sigma_A^2 + r\sigma_{AB}^2 + \sigma^2$, $r = 2$, $a = 3$, and $b = 4$.

Read off E(MSB) from the shorthand.