

## What is Probability?

- American Heritage Dictionary Definition 3: “*Math.* A number expressing the likelihood of occurrence of a specific event, such as the ratio of the number of experimental results that would produce the event to the total number of results considered possible.”
- AHD Definition 1 of Likelihood: “The state of being likely or probable; probability.”

## From the web

- “Probability is the name given to the branch of mathematics that deals with chance and how to predict whether a result is likely or unlikely.”  
(<http://www.learn.co.uk/default.asp?WCI=Unit&WCU=275>)
- “By probability, we generally mean the likelihood of a particular event occurring, given a particular set of circumstances. The probability of an event is generally expressed as a quantitative measurement.”  
(<http://www.maps.jcu.edu.au/hist/stats/quet/quet6.htm>)

## Compare:

- What is time?
- What is a point?

## Probability of an Event: Three Perspectives

- Classical (“A priori” or “theoretical”)
- Empirical (“A posteriori” or “Frequentist”)
- Subjective

## Classical Probability (“A Priori” or “Theoretical”)

- Situation: “experiment” or “random process” with  $n$  equally likely outcomes.
- E.g, toss a fair die: Six equally likely outcomes,
- $P(A) = m/n$ , where  $A$  is satisfied by exactly  $m$  of the  $n$  outcomes
- E.g., toss a fair die;  $A$  = an odd number comes up  
->  $P(A) = 3/6$ .

## Pros and Cons of Classical Probability

- Conceptually simple for many situations
- Doesn’t apply when outcomes are not equally likely.
- Doesn’t apply when there are infinitely many outcomes

### Empirical Probability (“A Posteriori” or “Frequentist”)

- $P(A) = \lim_{n \rightarrow \infty} (m/n)$ , where  $n$  = number of times process performed,  $m$  = number of times  $A$  is satisfied.
- E.g., toss a fair die;  $A$  = six lands up
- E.g., toss a die that is suspected of not being fair;  $A$  = six lands up.

### Pros and Cons of Empirical Probability

- Covers more cases than classical.
- Intuitively agrees with classical when classical applies.
- Repeating the identical experiment an infinite number of times (sometimes even twice) is physically impossible.
- How large must  $n$  be to give a good approximation to the limit?

### Subjective Probability

- A person’s measure of belief that some given event will occur.
- E.g.,  $P(\text{the stock market will go up tomorrow})$ .
- Needs to be “coherent” to be workable. (e.g.,  $P(\text{stock market goes up tomorrow}) = .6$  and  $P(\text{stock market goes down tomorrow}) = .7$  are inconsistent.)

### Pros and Cons of Subjective Probability

- Applicable in situations where other definitions are not.
- Fits intuitive sense of probability.
- Can be considered to extend classical.
- Can vary from individual to individual
- Requires “coherence” conditions; are people always that rational?

### Unifying Perspective: Axiomatic Model of Probability

A function  $P$  from events to non-negative numbers satisfying:

1.  $0 \leq P(E) \leq 1$
2.  $P(S) = 1$  ( $S$  = certain event; sample space)
3.  $P(\text{union of mutually exclusive events}) = \text{sum of } P \text{ of individual events}$