

To obtain the lowess estimates $\text{lowess} \pm \text{SD}$:

Notation: To distinguish estimates from parameters or true values, we'll use a "hat" (^) over estimates whenever possible.

1. Smooth to get the lowess estimate $\hat{E}(y|x)$ of $E(y|x)$.

2. Let

$$\hat{e} = y - \hat{E}(y|x) \text{ -- the "lowess residual"}$$

It estimates the "error" (deviation from the conditional mean) $e = y - E(y|x)$

Recalling the definition of Var, we can consider

$$E(\hat{e}^2|x)$$

as an estimate of $\text{Var}(y|x)$.

3. Use smoothing (lowess) to get an estimate $\hat{E}(\hat{e}^2|x)$ of $E(\hat{e}^2|x)$

$$4. \widehat{\text{SD}} = \sqrt{\hat{E}(\hat{e}^2|x)}$$