OPTIONAL SUPPLEMENT FOR CHAPTER 27: PROPERTIES OF THE INTERCEPT

Here are some properties of the least squares estimator (using b_0, by abuse of notation) for simple linear regression:

- b_0 is an unbiased estimator of β_0.
- Var (b_0) = $\sigma^2\left(\frac{1}{n}+\frac{\bar{x}^2}{SXX}\right)$, so
- SD(b_0) = $\sigma\sqrt{\frac{1}{n}+\frac{\bar{x}^2}{SXX}}$

Exercise: Analyze the variance formula to answer:

a. A larger $\bar{x}$ gives a ____________ variance for b_0.
   → Does this agree with intuition?

b. A larger sample size tends to give a ____________ variance for b_0.

c. If $\bar{x} \geq 1$, the variance of b_0 is _______________ than the variance of b_1.
   → Does this agree with intuition?

d. How does spread of the x_i's affects the variance of b_0?

- The covariance of b_0 and b_1 is

  $$\mathrm{Cov}(\hat{\eta}_0, \hat{\eta}_1) = -\sigma^2 \frac{\bar{x}}{SXX}$$

Thus:

- b_0 and b_1 are *not* independent (except possibly when ____________________)
  → Does this agree with intuition? (Using the Least Squares Demo or Data-Fitting Demo Using Least Squares and Least Absolute Value might be helpful in exploring this.)

- The sign of $\mathrm{Cov}(\hat{\eta}_0, \hat{\eta}_1)$ is opposite that of $\bar{x}$.
  → Does this agree with intuition?