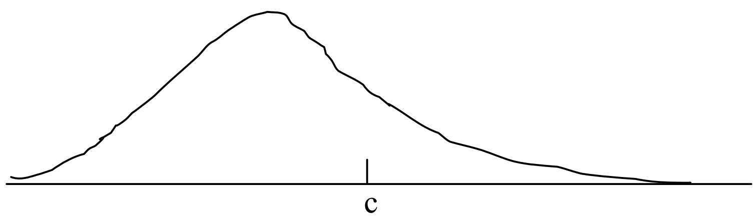
CUMULATIVE DISTRIBUTION FUNCTIONS

We've talked about the *probability density function* of a random variable. There is another function associated with a random variable that is often useful as well: the *cumulative distribution function* (cdf). The cdf FX of the random variable X is defined as

$$F_X(x) = P(X \leq x)$$

Exercises:

1. a. The diagram shows the graph of the pdf $f_X(x)$ of the *continuous* random variable X. How can you draw something in the picture that shows $F_X(c)$, the value of the cdf of X at c? [Hint: Remember the definition of the pdf.]



b. Use the idea in part (a) to give a formula for finding $F_X(c)$ in terms of $f_X(x)$ (still assuming X is a *continuous* random variable).

2. If X is a *continuous* random variable and you know its cdf, how can you find its pdf? [Hint: Use Exercise 1(b).]

3. If a < b, what can you say about the relationship between $F_X(a)$ and $F_X(b)$? (Your answer and reasoning should just depend on the definition of cdf, *not* on whether X is discrete or continuous.)

4. X is a discrete random variable that only takes on values 0, 1, 2, and 4, with probabilities ½, ¼, 1/8, and 1/8, respectively. What is the cdf of X? Sketch the cdf.

5. If X is a discrete random variable and you know the pdf f_X of X, how can you find the cdf F_X?

6. If X is a discrete random variable and you know the cdf, how can you find the pdf?

7. If X is a random variable and a and b are real numbers, then it makes sense to talk about the random variable aX + b: It involves the same random process as X, but has values calculated as aX + b. Let Y = aX +b for some constants a and b. (Assume $a \neq 0$.)

   i. If a > 0, express the cdf $F_Y(y)$ of Y in terms of the cdf F_X of X. Show each step in your reasoning.

   ii. The same, but now assume a < 0.

8. If X is a continuous random variable and Y is as in Exercise 7, find the pdf $f_Y(y)$ of Y in terms of the pdf f_X of X. [Hint: Exercises 2 and 7. Also, be careful if a is negative.]

9. If X and Y are as in Exercise 8, find E(Y) in terms of E(X). [Hint: Use Exercise 8 and the definition of expected value. Be careful when $a < 0$.] (Note: This is also true for X discrete, but I won't ask you to prove it.)

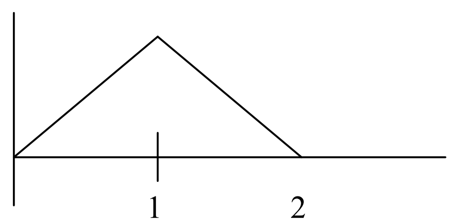
10. [*Note*: You may *not* use any prior knowledge you might have about normal distributions in doing this problem; you may only use things that have been in the handouts in this class so far.] If Z is the standard normal random variable and $Y = aZ + b$ (where $a \neq 0$):

   a. Use Exercise 8 to find the pdf of Y.
   b. Using the result of part (b), what kind of random variable is Y? Explain.
   c. Using part b and Exercise 9, plus the fact that $E(Z) = 0$, find the mean of a normal random variable with parameters μ and σ.

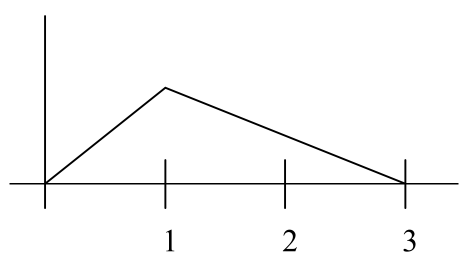
11. Use the idea in Exercise 1(a) to help you *sketch* the cdf of each of the following random variables. In other words, do this by reasoning "qualitatively" rather than working with formulas.

a. Uniform on (A,B).

b. Graph of pdf is



c. Graph of pdf is



d. A normal distribution.