WHAT DO YOU MEAN BY AVERAGE?

I. *Warm-up Exercise*: ***->Be sure to do this problem first.***
1. (For discussion and reflection; not to hand in.) The first two columns of the following table give miles-per-gallon (MPG) for five pairs of cars. For each pair, the car whose MPG is in the first column is less fuel-efficient than the car in the second column. Consider replacing the first car of a pair with the second car of the same pair. Rank these five swaps in order of their benefit to the environment (assuming they all travel the same number of miles, under the same conditions). Use 1 for most beneficial, 5 for least. *What reasoning did you use in determining your ranking?*

	MPG of old car	MPG of new car	Estimated Rank of benefit of replacing old car with new
First pair	34	50	
Second pair	18	28	
Third pair	42	48	
Fourth pair	16	20	
Fifth pair	22	24	

II. *We have discussed how different measures might be appropriate for different circumstances. One type of measure frequently used is a "measure of center." You are probably familiar with the three most common measures of center: mean, median and mode. In the next problem, we explore the concept of "mean" and "average" in different contexts.*

Often in mathematics, we use the same word with two different meanings. One place this might cause confusion is when we talk about "average speed". If we are given speeds $v_1, v_2, \ldots, v_n$, then "average speed" might be interpreted as the average of the n numbers – that is, $(v_1 + v_2 + \ldots + v_n)/n$. But we also use "average speed" to mean (total distance traveled)/(total time traveled). *To avoid confusion, let us agree to use "average of the speeds" for the average of the numbers, and "average speed" for distance/time.*

2. *Average Speeds Problem* → Hand in parts (a) – (d) *Reminder*: You are expected to give your reasons and write up your solutions clearly.

a. Suppose you travel for *length of time t* at speed v_1, then travel for the same length of time t at speed v_2, and so forth, finishing by traveling the same length of time t at speed v_n. What is your average speed for the whole trip? How does the expression you get compare with the expression for the average of the speeds (or the average of any other set of numbers)?

b. Now suppose you travel for a_1 intervals of time t at speed v_1, then travel a_2 intervals of time t at speed v_2, and so forth, finishing by traveling a_n intervals of time t at speed v_n. What is your average speed for the whole trip? How does the expression you get compare with the expression for the average of the speeds (or the average of any other set of numbers)?

c. Now suppose you travel *distance d* at speed v_1, then travel the same distance d at speed v_2, and so forth, finishing by traveling the same distance d at speed v_n. What is your average speed for the whole trip? How does the expression you get compare with the expression for the average of the speeds? (or the average of any other set of numbers)?

d. Now suppose you travel a_1 lengths of distance d at speed v_1, then travel a_2 lengths of distance d at speed v_2, and so forth, finishing by traveling a_n lengths of distance d at speed v_n. What is your average speed for the whole trip? How does the expression you get compare with the expression for the average of the speeds? (or the average of any other set of numbers)?

e. According to the *CAFÉ Overview – Frequently Asked Questions* website of the National Highway Traffic Safety Administration (http://www.nhtsa.dot.gov/cars/rules/cafe/overview.htm), the *corporate average fuel economy* (*CAFÉ*) is " the sales weighted average fuel economy, expressed in miles per gallon (mpg), of a manufacturer's fleet of passenger cars or light trucks with a gross vehicle weight rating (GVWR) of 8,500 lbs. or less, manufactured for sale in the United States, for any given model year." The actual formula used for calculating the CAFÉ is

$$\text{CAFÉ} = \frac{1}{\frac{w_1}{f_1} + \frac{w_2}{f_2} + \ldots + \frac{w_n}{f_n}},$$

where n is the number of types of cars and trucks in the manufacturer's "fleet", f_i is the fuel economy of vehicle type i (in mpg = miles per gallon), and w_i is the proportion of the total vehicles sold by the manufacturer that are of type i.

i. What are the units of CAFÉ?

ii. Explain why this is a reasonable measure of "the sales weighted average fuel economy" of the fleet. (Hint: Some of parts (a) - (d) might give you some ideas.)

iii. What might be the advantages and disadvantages of CAFE as a measure of the manufacturer's attention to fuel economy?

3. a. A few years ago, the "puzzler" on the NPR radio program "Car Talk" (Click and Clack) involved a couple who had two vehicles. One was a gas-guzzling SUV that got 10 mpg; the other was a super efficient hybrid that got 100 mpg. Each car was driven the same number of miles each year. The couple wanted to improve the average number of miles per gallon for their household. Only two options were available: either tune up the SUV to get 11 mpg, or replace the hybrid with a super hybrid that would get 200 mpg. Figure out which choice would be best to achieve their goal.

b. Now look again at Problem 1 (the warm-up exercise). Do you agree with your original ranking and reasoning or would you like to change it?

c. Per passenger fuel consumption for commercial air travel is about 70MPG. Which would be more fuel efficient: eliminating 10,000 miles of flying or replacing a 33 MPG car that travels 10,000 miles per year with a car that gets 50 MPG?

d. Many countries using the metric system measure fuel consumption in liters per 100 kilometers, rather than kilometers per liter. What might be the advantage of using a similar system (e.g. gallons per 100 miles instead of miles per gallon) in the U.S.?