M340L First Midterm Exam, February 18, 2010

1.Let $A = \begin{pmatrix} 1 & 2 & 5 & 0 \\ 2 & 1 & 7 & 6 \\ 3 & 3 & 12 & 6 \end{pmatrix}$.

a) Compute A_{rref}.

b) Find all solutions to $A\mathbf{x} = 0$.

c) Find all solutions to $A\mathbf{x} = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$.

d) Find all solutions to $A\mathbf{x} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$.

2. Consider the vectors $\mathbf{a}_1 = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, $\mathbf{a}_2 = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$, $\mathbf{a}_3 = \begin{pmatrix} 5 \\ 7 \\ 12 \end{pmatrix}$, $\mathbf{a}_4 = \begin{pmatrix} 0 \\ 6 \\ 6 \end{pmatrix}$, $\mathbf{u} = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$, and $\mathbf{v} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$.

a) Are the vectors $\{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3, \mathbf{a}_4\}$ linearly independent? If not, write the zero vector as a nontrivial linear combination of $\{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3, \mathbf{a}_4\}$.

b) Is $\mathbf{u}$ in the span of $\{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3, \mathbf{a}_4\}$? If so, write $\mathbf{u}$ as a linear combination of $\{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3, \mathbf{a}_4\}$.

c) Is $\mathbf{v}$ in the span of $\{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3, \mathbf{a}_4\}$? If so, write $\mathbf{v}$ as a linear combination of $\{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3, \mathbf{a}_4\}$.

3. (a) Let $T_1 : R^2 \to R^2$ be a linear transformation that first rotates vectors 90 degrees counterclockwise and then reflects them across the vertical axis. Find the (standard) matrix of T_1. Is T_1 1-1? If T_1 onto?

(b) Let $T_2 : R^2 \to R^3$ be given by the formula $T_2 \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 3x_1 - x_2 \\ 0 \\ 9x_1 - 3x_2 \end{pmatrix}$.

Find the (standard) matrix of T_2. Is T_2 1-1? If T_2 onto?

4. (a) Does $\begin{pmatrix} 2 & 3 \\ 6 & 9 \end{pmatrix}$ have an inverse? If so, find it.

(b) Does $\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 6 \end{pmatrix}$ have an inverse? If so, find it.

5. True/false. Just mark each statement with a T (or TRUE) or an F (or FALSE). You do not need to justify your answers, and partial credit will not be given.

a) If A is a left-inverse, then the columns of A are linearly independent.

b) If a 3×5 matrix A has rank 3, then the linear transformation $T(\mathbf{x}) = A\mathbf{x}$ is onto.

c) If $A\mathbf{x} = 0$ has infinitely many solutions, then $A\mathbf{x} = \mathbf{b}$ has infinitely many solutions.

d) If $T(\mathbf{x}) = A\mathbf{x}$, then the third column of A (assuming A has at least three columns, of course) is $T(\mathbf{e}_3)$.

e) If A is a 3×5 matrix, then the columns of A are linearly dependent.

f) If A and B are row-equivalent matrices, then the equations $A\mathbf{x} = \mathbf{b}$ and $B\mathbf{x} = \mathbf{b}$ have the same solutions.

g) The columns of $\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & 8 \\ 1 & 3 & 9 & 27 \\ 1 & 4 & 16 & 64 \\ 1 & 5 & 25 & 125 \end{pmatrix}$ span R^5.

h) If four vectors in R^4 are linearly independent, then they span R^4.

i) If a square matrix has a right-inverse, then the columns are linearly independent.

j) If A and B are 2×2 matrices, then $AB = BA$.