

Some setup: commands for plotting the dynamical system (ignore this.)

```
In[275]:= iterates[x10_, x20_, L_, S_] :=  
  Partition[Flatten[{x[0] := {{x10}, {x20}};  
    x[n_] := S.x[n - 1]; Table[x[n], {n, 0, L}]}], 2]
```

```
In[276]:= multiiterates[x0s_, L_, S_] := Flatten[  
  Table[iterates[x0s[[i, 1]], x0s[[i, 2]], L, S], {i, 1, Length[x0s]}], 1]
```

```
In[277]:= plotmultiiter[x0s_, L_, S_] := Show[  
  ListPlot[multiiterates[x0s, L, S], PlotStyle -> PointSize[Large],  
    AspectRatio -> 1, PlotRange -> {{-5, 5}, {-5, 5}}, ListPlot[  
    x0s, PlotStyle -> {Red, PointSize[Large]}, AspectRatio -> 1]]
```

Constructing a diagonal matrix B. (In lecture this matrix was called D, but *Mathematica* reserves the name "D" for the differentiation operator.)

```
In[278]:=  $\lambda_1 = 1.14; \lambda_2 = 0.87;$ 
```

```
In[279]:= B = {{ $\lambda_1$ , 0}, {0,  $\lambda_2$ }};
```

```
In[280]:= B // MatrixForm
```

Out[280]/MatrixForm=

$$\begin{pmatrix} 1.14 & 0 \\ 0 & 0.87 \end{pmatrix}$$

A matrix A which is similar to B; the two are related by the change-of-basis matrix P.

```
In[281]:= P = {{1, 1}, {-1, 1}};
```

```
In[282]:= A = P.B.Inverse[P];
```

```
In[283]:= A // MatrixForm
```

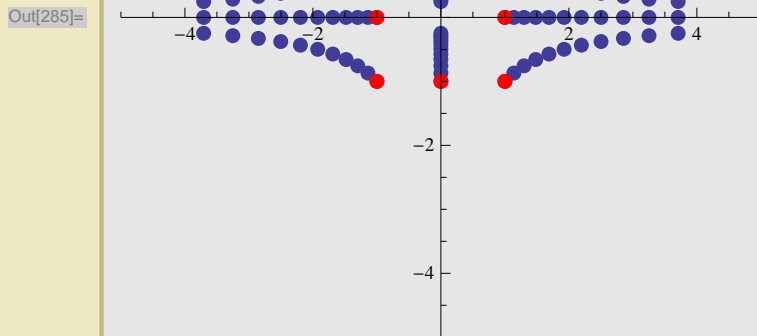
Out[283]/MatrixForm=

$$\begin{pmatrix} 1.005 & -0.135 \\ -0.135 & 1.005 \end{pmatrix}$$

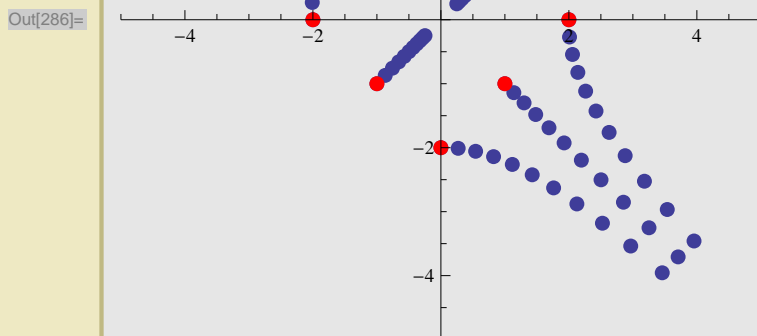
Iterating the dynamical systems defined by B (first) and A (second) for 10 time steps, with a few different choices of initial condition. The initial points are shown in red. The two pictures are related to one another by the linear transformation P.

```
In[284]:= startingpoints = {{1, 0}, {-1, 0},  
  {0, 1}, {0, -1}, {1, 1}, {1, -1}, {-1, -1}, {-1, 1}};
```

```
In[285]:= plotmultiiter[startingpoints, 10, B]
```



```
In[286]:= plotmultiiter[Transpose[P.Transpose[startingpoints]], 10, A]
```



The eigenvalues of A. Note they are the same as the eigenvalues of B, as they must be since the two matrices are similar.

```
In[287]:= Eigenvalues[A]
```

```
Out[287]= {1.14, 0.87}
```

```
In[288]:= Eigenvalues [B]
```

```
Out[288]= {1.14, 0.87}
```

The absolute values of the eigenvalues. These predict whether the dynamical system has an attracting, repelling or saddle point.

```
In[289]:= Abs [Eigenvalues [A] ]
```

```
Out[289]= {1.14, 0.87}
```