M 361K Spring 2011 (55380), Sample Midterm

Name: ______________________________

Question	Points	Score
1	60	
2	40	
3	20	
Total:	120	

Problem 1 (60 points).

(a) (20 points) State the definition of convergence.

(b) (20 points) Prove that the sequence $a_n = 1/(n^4 + 1)$ converges to 0. Assume only the definition of convergence and the Archimedean property of the real numbers (for any $x \in \mathbb{R}$, there exists some $n \in \mathbb{N}$ such that $n > x$.)

(c) (20 points) Suppose that a_n is a sequence which converges to $L \in \mathbb{R}$ and also converges to $M \in \mathbb{R}$. Prove that $L = M$. Assume only the definition of convergence.

Problem 2 (40 points).

True or False. If the statement is true, give a proof. If the statement is false, give a counterexample and *prove* that it is a counterexample. You may freely use any statement which was proved in the homework, *except* for the precise statement you are being asked to prove.

(a) (20 points) If the sequence a_n is divergent, and all $a_n \neq 0$, then the sequence $b_n = 1/a_n$ is also divergent.

(b) (20 points) If the sequence a_n converges to L, then the sequence $b_n = |a_n|$ converges to $|L|$.

Problem 3 (20 points).

For this question you may freely use any fact proven in the homework, *except* for the precise statement you are being asked to prove.

(a) (20 points) Prove that if $r > 1$ then the sequence $a_n = r^n$ diverges to $+\infty$. You need not write out every detail; just explain clearly the main steps.