M 365C

Fall 2013, Section 57465

Midterm 1 samples

True or False. If true, sketch a proof in a few lines. If false, state a counterexample. Throughout, let X denote a metric space.

1. If $E \subset X$ is compact, then E^c is open.
2. If $E \subset \mathbb{R}$ is countable, then E is closed.
3. If $E \subset X$, and $\epsilon > 0$, then $\bigcup_{p \in E} N_\epsilon(p)$ is open.
4. If E° is open, then E is open. (Recall that E° is the set of all interior points of E.)
5. The sequence $\{p_n\}$ in X converges if and only if every subsequence of $\{p_n\}$ converges.
6. Given any collection of closed intervals in $\mathbb{R}$, the union of the collection is closed.
7. Every bounded subset of $\mathbb{R}$ is contained in a compact set.
8. Every nonempty compact subset of $\mathbb{R}$ has a limit point.
9. If $E \subset \mathbb{R}$ is bounded, then $\{a + b \mid a, b \in E\}$ is also bounded.
10. Every subset of $\mathbb{Q}$ which is bounded below in $\mathbb{Q}$ has a greatest lower bound in $\mathbb{Q}$.
11. Every subset of $\mathbb{Q} \subset \mathbb{R}$ which is bounded below in $\mathbb{Q}$ has a greatest lower bound in $\mathbb{R}$.
12. If $E \subset X$ is disconnected, then $\bar{E}$ is also disconnected.
13. If $K \subset X$ is compact and $p \in X$, the set $\{d(p, q) \mid q \in K\}$ has a minimum element.
14. If the sequence $\{p_n\}$ in $\mathbb{R}$ converges, then the sequence $\{p_n^2\}$ also converges.
15. If the sequence $\{p_n^2\}$ in $\mathbb{R}$ converges, then the sequence $\{p_n\}$ also converges.
16. If $|p_n| < 2$ for all n, then the sequence $\{p_n\}$ has a convergent subsequence.