Nonabelian gauge theory

Now, replace $U(1)$ by a general compact Lie group G:

$\mathcal{C} = \{ (P, \nabla) \ G\text{-bundle w/ conn. over } X \}$ $\quad \mathcal{G} = \{ \text{equivalences } (P,\nabla) \sim (P',\nabla') \}$

Fix invariant bilinear form (normalized to be 2 on the roots)

$$S = \frac{1}{4g^2} \int_X \mathrm{Tr}(F \wedge * F) + \frac{\theta}{8\pi^2} \int \mathrm{Tr}(F \wedge F)$$

Unlike the abelian theory, this is already <u>interacting</u>:

in triv. $\nabla = d + A$, we have $F = dA + [A,A]$

so there are both <u>cubic and quartic</u> interactions.

$\dim A = 1.$
$\dim F = 2.$

Have to analyze it by RG methods like we did for the interacting scalar.

Only <u>two</u> func. on $\mathcal{C}/\mathcal{G}$ with $\dim \leq 4$, namely $\mathrm{Tr}(F \wedge * F)$ and $\mathrm{Tr}(F \wedge F)$.

Now, a <u>crucial difference</u> from scalar field theory: this theory has $\Lambda \frac{dg}{d\Lambda} = -\frac{11N}{48\pi^2} g^3 + \cdots$

(for $G = SU(N)$)

So, coupling gets <u>weaker</u> as Λ grows.

Perturbation theory gets <u>more</u> reliable. "asymptotic freedom" (Nobel Prize 2004 Gross–Politzer–Wilczek)

Generally believed that in this case it's safe to take $\Lambda_0 \to \infty$: the theory is "UV complete"

Conversely, coupling gets <u>stronger</u> at lower energies.

Let Λ_s be the energy at which $g = 1$.

It's generally believed that the theory is actually <u>trivial</u> at $E \ll \Lambda_s$.

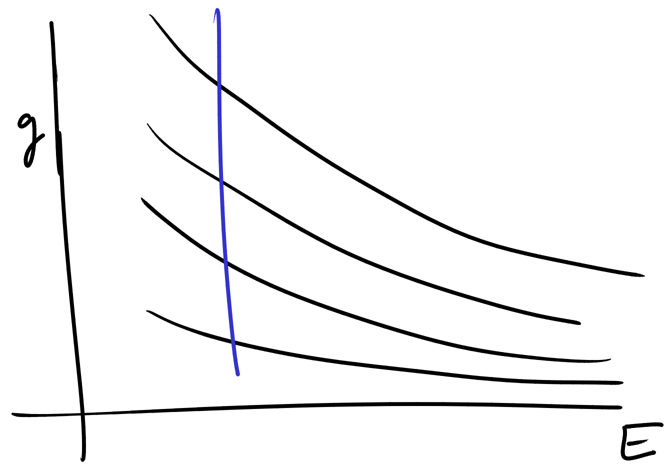
[Concrete consequence: e.g. $\langle \mathrm{Tr}\, F^2(x)\, \mathrm{Tr}\, F^2(0) \rangle \sim e^{-c\Lambda_s x}$.]

Radically different from what we'd get in <u>perturbation theory</u>: $D(x,0) \sim \frac{1}{x^2}$

No real proof even by physics standards. But, agrees w/ <u>observation</u>: we don't see $SU(3)$ <u>gauge fields</u> around...

It is worth \$1M from the Clay Foundation to rigorously (formulate and) prove this!

<u>RG trajectories:</u>



Can be fixed either by specifying g at some fixed E,
or by specifying the scale Λ_s. (The latter is usually preferred.)
