Standard Model

Gauge theory with $G = SU(3) \times SU(2) \times U(1)$

Higgs field ϕ charged under $SU(2) \times U(1)$: if $\phi \sim m \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ then

$$Z_\phi = \left\{ \left(g, \begin{pmatrix} e^{i\alpha} & 0 \\ 0 & e^{-i\alpha} \end{pmatrix}, e^{-i\alpha} \right) \right\} \subset SU(3) \times SU(2) \times U(1)$$

is $U(1)$

Following the same procedure we followed in simpler examples here gives terms in S of the form (unprimed for $SU(2)$, primed for $U(1)$)

$$\frac{m^2}{8}\left[g^2 \left(\|A^1\|^2 + \|A^2\|^2 \right) + \|gA^3 - g'A'\|^2 \right] \quad \left(+ \frac{1}{2}\|F\|^2 + \frac{1}{2}\|F'\|^2 \right)$$

Diagonalizing this then gives

$$W^\pm = \frac{1}{\sqrt{2}}\left(A^1 \mp iA^2\right)$$

$$Z = \frac{1}{\sqrt{g^2 + g'^2}}\left(gA^3 - g'A'\right)$$

$$m_W = \frac{1}{2} g m$$

$$m_Z = \frac{1}{2}\sqrt{g^2 + (g')^2}\, m$$

$$\left[\text{from diagonalizing } \begin{pmatrix} g^2 & -gg' \\ -gg' & (g')^2 \end{pmatrix} \right]$$

Measurements up to ~ TeV so far

$\sim 2 \cdot 10^{-33} g$

eV	KeV	MeV	GeV	TeV
1	10^3	10^6	10^9	10^{12}

Scales:

- electron mass: 511 KeV
- quark masses: u, d: ~1 MeV, s ~100 MeV, c~1 GeV, b~4 GeV, t~170 GeV
- weak scale: ~ 100 GeV ($W^\pm \sim 80$ GeV, $Z \sim 90$ GeV)

At 100 GeV, $\alpha_{SU(2)} \sim \frac{1}{30}$ $\left[\alpha = \frac{g^2}{4\pi} \right]$