

Forms on almost \mathbb{C} manifolds

Notation $\Omega^k(M) = C^\infty$ sections of $\wedge^k(T^*M)$ over M .

Recall the exterior derivative operator $d: \Omega^k(M) \rightarrow \Omega^{k+1}(M)$.

$$d(f dx_{i_1} \wedge \dots \wedge dx_{i_k}) = \sum_{j=1}^m \frac{\partial f}{\partial x_j} dx_j \wedge dx_{i_1} \wedge \dots \wedge dx_{i_k}$$

$$d(dw) = 0$$

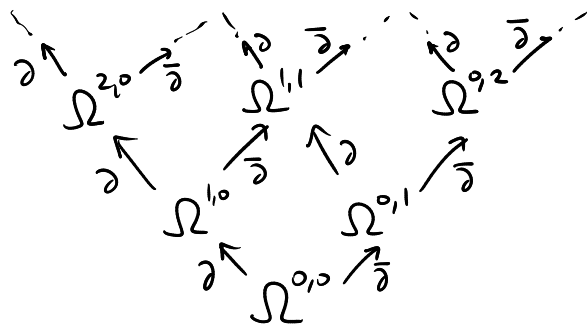
Notation For M almost complex, $\wedge^k(T^*M) = \bigoplus (T^*)^{p,q} M$,
and let $\Omega^{p,q}(M)$ be the space of C^∞ sections of $(T^*)^{p,q} M$.

Thus $\Omega^k(M) = \bigoplus_{p+q=k} \Omega^{p,q}(M)$.

Def Let $\Pi^{p,q}$ be projection on the $\Omega^{p,q}$ summand.

Let $\partial: \Omega^{p,q}(M) \rightarrow \Omega^{p+1,q}(M)$ be $\Pi^{p+1,q} \circ d$.

Let $\bar{\partial}: \Omega^{p,q}(M) \rightarrow \Omega^{p,q+1}(M)$ be $\Pi^{p,q+1} \circ d$.



Ex If $\omega = z_1 z_2 dz_2 + z_1 \bar{z}_2 dz_1 \in \Omega^{1,0}(\mathbb{C}^2)$

$$\text{then } d\omega = \underbrace{z_2 dz_1 \wedge dz_2}_{\partial\omega \in \Omega^{2,0}} + \underbrace{z_1 d\bar{z}_2 \wedge dz_1}_{\bar{\partial}\omega \in \Omega^{1,1}}$$

Prop $\partial(\alpha \wedge \beta) = \partial\alpha \wedge \beta + (-1)^{|\alpha|} \alpha \wedge \partial\beta$, $\bar{\partial}(\alpha \wedge \beta) = \bar{\partial}\alpha \wedge \beta + (-1)^{|\alpha|} \alpha \wedge \bar{\partial}\beta$

Pf If $\alpha \in \Omega^{p,q}$, $\beta \in \Omega^{p',q'}$ then $d(\alpha \wedge \beta) = d\alpha \wedge \beta + (-1)^{|\alpha|} \alpha \wedge d\beta$
then project on $\Omega^{p+p'+1, q+q'}$ in each term.