Last time: \( \lim_{x \to a} f(x) = L \)
\( \lim_{x \to a} f(x) = \infty \)

Example: \( \lim_{x \to 0} \frac{1}{x^2} = \infty \)

Q 1) What is \( \lim_{x \to -3} \frac{x}{(x+3)^2} \)?
- \( -\infty \), DNE, 0

Plug in \( x = -3 \):
\( \frac{-3}{(-3+3)^2} = \frac{-3}{0} \to \text{no help} \)

If \( x \) is close to 3:
\( \frac{x}{(x+3)^2} = \frac{\text{close to 3}}{\text{very small positive}} \)
\( = \frac{\text{very large negative}}{} \)

So, \( \lim_{x \to -3} \frac{x}{(x+3)^2} = -\infty \)

Q 2) What is \( \lim_{x \to 2} \frac{x-2}{x^2-4} \)?

Plug in \( x = 2 \):
\( \frac{2-2}{4-4} = \frac{0}{0} \to \text{no help} \)

But, \( \lim_{x \to 2} \frac{x-2}{x^2-4} = \lim_{x \to 2} \frac{x-2}{(x-2)(x+2)} \)
\( = \lim_{x \to 2} \frac{1}{x+2} = \frac{1}{4} \)

Ex: \( \lim_{x \to 1} \frac{1}{x-1} \)

If \( x \) is slightly bigger than 1, \( x = 1.00001 \)
then \( \frac{1}{x-1} = \text{big positive} \)

If \( x \) is slightly smaller than 1, \( x = 0.99999 \)
then \( \frac{1}{x-1} = \text{big negative} \)

So,
\( \lim_{x \to 1^+} \frac{1}{x-1} = +\infty \)
\( \lim_{x \to 1^-} \frac{1}{x-1} = -\infty \)
\[
\lim_{x \to 1} \frac{1}{x-1} \text{ DNE!}
\]

\[
\text{Ex: } \lim_{x \to 0^+} \sin \left(\frac{1}{x}\right)
\]

DNE.

**Limit Laws**

Suppose \( c \) is any constant, and the limits \( \lim_{x \to a} f(x) \) and \( \lim_{x \to a} g(x) \) exist.

Then:

1. \( \lim_{x \to a} (f(x) + g(x)) = \lim_{x \to a} f(x) + \lim_{x \to a} g(x) \)
2. \( \lim_{x \to a} (f(x) - g(x)) = \lim_{x \to a} f(x) - \lim_{x \to a} g(x) \)
3. \( \lim_{x \to a} c \cdot f(x) = c \cdot \lim_{x \to a} f(x) \)
4. \( \lim_{x \to a} f(x) \cdot g(x) = (\lim_{x \to a} f(x))(\lim_{x \to a} g(x)) \).

**Ex:** if \( \lim_{x \to 3} f(x) = 7 \), \( \lim_{x \to 3} g(x) = 8 \) then \( \lim_{x \to 3} f(x)g(x) = 56 \).

5. \( \lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)} \) (if \( \lim_{x \to a} g(x) \neq 0 \))

6. \( \lim_{x \to a} f(x)^n = (\lim_{x \to a} f(x))^n \)

**Ex:**

\[
\lim_{x \to 0} \frac{x^2}{3 \sin^2 x} = \frac{1}{3} \lim_{x \to 0} \frac{x^2}{\sin^2 x}
\]

\[
= \frac{1}{3} \left( \lim_{x \to 0} \frac{x}{\sin x} \right)^2
\]

\[
= \frac{1}{3} \left( \lim_{x \to 0} \frac{1}{x} \right)^2
\]

\[
= \frac{1}{3} \left( \frac{1}{1} \right)^2 = \frac{1}{3}
\]

OC. \[
\frac{1}{3} \left( \lim_{x \to 0} \frac{\sin x}{x} \right)^{-2}
\]

\[
= \frac{1}{3} \cdot 1^{-2}
\]

\[
= \frac{1}{3}
\]
\( \lim_{x \to a} c = c \) 

\( \lim_{x \to a} x = a \) 

\( \lim_{x \to a} x^a = a^a \) 

\( \lim_{x \to a} \sqrt{x} = \sqrt{a} \) 

\( \lim_{x \to a} \sqrt{f(x)} = \sqrt{\lim_{x \to a} f(x)} \)

\( \lim_{x \to 0} x^2 + 9 = ? \) 

\[ = 9 \]

\( \lim_{x \to 0} \sqrt{x^2 + 9} = ? \) 

\[ = \sqrt{\lim_{x \to 0} x^2 + 9} \] 

(by Limit Law 11)

\[ = \sqrt{\lim_{x \to 0} x^2 + \lim_{x \to 0} 9} \]

\[ = \sqrt{0^2 + 9} \]

\[ = \sqrt{9} = 3 \]

\( \lim_{x \to 0} \frac{\sqrt{x^2 + 9} - 3}{x^2} = ? \) 

\quad \text{ phymin 0: } \frac{3 - 3}{0^2} = \frac{0}{0} \quad \text{ no help} 

Simplify: 

\[ \lim_{x \to 0} \frac{\sqrt{x^2 + 9} - 3}{x^2} \cdot \frac{\sqrt{x^2 + 9} + 3}{\sqrt{x^2 + 9} + 3} = \lim_{x \to 0} \frac{x^2 + 9 - 9}{x^2(\sqrt{x^2 + 9} + 3)} \]

\[ = \lim_{x \to 0} \frac{x^2}{x^2(\sqrt{x^2 + 9} + 3)} \]

\[ = \lim_{x \to 0} \frac{1}{\sqrt{x^2 + 9} + 3} = \frac{1}{6} \]